Exploring Heterogeneity for Cooperative Localization in Swarm Robotics

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***Abstract*—Cooperative localization allows groups of robots to improve their overall localization by sharing position estimates within the team. In Swarm Robotics a large number of very simple agents is used to perform different types of tasks, however, this simplicity may have a direct impact on the estimated localization. In this work, we consider the use of a single robot (leader) with improved localization capability (e.g. GPS) which will be used to enhance the position estimates of the rest of the group. By using a potential function, we are able to place the leader in the region (near the center) that best benefits its position broadcasting and also to execute a coordinated and continuous movement of the entire group by controlling only this unity. Numerous trials in a simulated environment were executed, providing statistical examination of the final results.**

I. Introduction

Robotic Swarms have been used in different types of tasks, such as covering large areas, search-and-rescue and patrolling ([1], [2]). In some of them, the swarm should navigate to specific locations while maintaining its cohesion to perform a given task. Many of these tasks are performed in places deprived of global localization systems. Consequently, it is necessary to provide the group with some cooperative capability to allow each robot to track its initial localization information.

Cooperative Localization (CL) techniques have been applied in various situations to provide a way of tracking the localization of a group of robots. These methods allow each robot to use other robots in the group as beacons or landmarks in order to update its own pose belief. A way to do this is to generate a joint estimate of all robots' poses in the group, by getting their individual pose estimations. In spite of possibly generating optimal estimates, this strategy needs to use all robots in the group, making it unfeasible when dealing with a large group of robots. To deal with this, approximate cooperative localization techniques have been employed in such a way that each robot uses only the pose of a part of the group to update its belief.

In [3], we have shown that it is possible to reach good individual localization levels when the robots use a limited number of neighbors as landmarks. For this, some members of the group must be kept stationary while the others are in motion. By doing this, the static robots can maintain low levels of uncertainty and act as localization providers. A group coordination mechanism instructed the robots to change their roles (stationary and in motion) allowing the group to advance in the environment while maintaining the uncertainty bounded. In spite of the good results, this strategy may compromise the group performance due to the intermittent movement.

In systems where the robots move continually, the quality of localization (position tracking) generally degrades with the distance traveled. This happens because of the lack of static landmarks providing good localization information. In this paper, we investigate the use of heterogeneous robots to perform cooperative localization in a swarm. A robot with global localization capabilities is placed inside the swarm and is used to provide good localization estimates and to guide the others during the continuous motion. The group movement is done in a cohesive way and only local sensing is used to maintain the group together. Furthermore, each robot uses only the neighbor with the lowest uncertainty as a landmark to improve its own localization estimates. The use of only a small set of neighbors for cohesion and localization makes this method scalable, which is a fundamental requirement when dealing with swarms. The results show that the method is capable of propagating good localization estimates through the group such that all robots in the swarm can reach good individual localization estimates during navigation.

This paper is organized as follows. Next section discusses some related works regarding cooperative localization. Section III presents our methodology, describing the mechanisms used for positioning the heterogeneous robots, keeping cohesion during navigation and performing the cooperative localization. Simulations are presented in Section IV, and Section V brings the conclusion.

II. Related Work

In multi-robot teams, individual localization estimates can be corrected based on the teammates' positions instead of environment landmarks. The first works to use robots as landmarks to perform cooperative localization are [4] and [5]. In these works, the motion of the robots is coordinated in such a way that in each moment a group of robots is stationary in order to serve as landmarks for others. These stationary robots observe (or are observed by) the others, promoting a more accurate position estimate than the one provided by proprioceptive sensors (e.g. odometry) only.

A more general approach to the cooperative localization problem is presented in [6], in which a *joint* estimate of the robots' poses in a group is generated by implementing an Extended Kalman Filter (EKF) [7]. The navigation of all robots is not coordinated and they are able to move continuously. The robots act individually and exchange information when they meet each other. In the decentralized approach, each robot performs the prediction step of the filter individually while the update step is done by exchanging information with others via communication and exteroceptive sensors. The localization interdependence is considered and its representation (cross-correlation terms) is stored by all robots and explicitly propagated to the teammates. Using these terms, a robot is capable of estimating its pose by considering the shared knowledge associated with previous meetings. In spite of having the best estimate, this strategy has the disadvantage of requiring a previous knowledge about the group size and presents a complexity that precludes its use for large groups of robots, such as swarms.

To deal with this drawback, some works have proposed approximate strategies to perform the belief update, such as [8] and [9]. These approaches use only part of the group to calculate the robot's estimate and propose strategies to deal with the partial knowledge. In [9] the belief update is performed by using the Covariance Intersection Algorithm (CI) [10], which is a consistent method to fuse estimates of same quantity with unknown cross-correlation terms. The approach presented in [9] allows each robot to maintain only its own state-covariance estimate with a cost to generate a new estimate of $O(n)$. The concept of *dependency tree* is introduced in [8], which prevents a circular reasoning in the update step of the beliefs. These two approaches avoid the overly optimistic position estimates that occurs when the localization interdependence is ignored.

In spite of these and other works in the context of cooperative localization, its use in robotic swarms is still incipient. In large groups of robots, the complexity in space and time necessary to deal with the complete measurement graph in order to generate a joint estimate presents a challenge in the area. In a swarm, occlusions are very common and limit the robots' observations. Moreover, the robots have limited capabilities that constrains each one in dealing with only local information.

Very few works have investigated the influence of the group size in relation to the quality of the group localization. The evidences presented in [3] show that good individual localization levels can be reached even when robots use a limited number of neighbors as landmarks. These evidences are supported by a coordinated motion strategy that let some robots with accurate beliefs to act as static landmarks. However, the localization quality is preserved at the expense of motion performance.

Many works have exploited the use of heterogeneous sensors aboard some team members of a group. By cooperatively sharing sensor information, the robots with limited capabilities have their shortages counterbalanced by their teammates. In [11], for example, a group of two robots equipped with different absolute positioning sensors assist each other in situations where the sensors' information have deteriorate or failed. When a failure occurs, the group performs cooperative localization. Another example is presented in [12], where a group of heterogeneous robots perform a 3D mapping task: a more specialized robot (master) controls two simpler ones (slaves) and use them to perform cooperative localization. In this way, the master robot is able to scan its surroundings and, using its accurate location, to transform the information caught by the laser range finder to the world coordinates and map the environment. On another approach, Bailey et al. [13] demonstrate that it is possible to transfer pose information from a well-localized robot to others that do not have localization capabilities. They use three robots, being one with sensors to measure absolute positioning, and all of them can share information with the others. Thus, a robot can directly access the information provided by the robot with accurate localization. These works show the advantages of using heterogeneous robots, but the scope of them is restricted to small groups of robots that can share localization information with all members of the group.

In this work we propose a methodology to tackle the aforementioned problems related to cooperative localization in swarms. Our methodology considers a single robot (leader) with improved localization capability (e.g. GPS) which will be used to enhance the position estimates of the rest of group. A motion strategy is also presented, which allows the swarm to execute a coordinated and continuous movement by controlling only the leader.

III. Methodology

Our methodology can be summarized as follows: initially, a leader robot, which has a good position estimate (for example, through the use of a Global Position System (GPS)), is autonomously positioned near the center of the swarm establishing an initial formation, as will be explained in Section III-A. At this moment, we consider that each robot has access to a good estimate of its initial position in a common frame and uses this initial information to start the motion. The leader has knowledge of the position of some waypoints that it uses to guide the group in the environment. The entire group is able to navigate as a unit by using the swarm motion strategy described in the Section III-B. Finally, during the motion, each robot uses proprioceptive sensors to estimate its pose. When the belief of a robot accumulates a certain amount of uncertainty, it tries to update such belief by using some of the robots inside its neighborhood as landmarks. The process is completely decentralized: each robot estimates its relative distance and orientation with respect to a landmark robot, and uses the pose information disseminated by it to correct its own pose (see Section III-C). In this way, the more accurate estimates of the leader are propagated through the swarm, allowing individual members to have a better estimation of their own pose.

We consider that robots are able to identify and measure relative ranges and bearings to their neighbors and exchange information with them. Also, robots are equipped with proprioceptive sensors that allow them to measure their own motion. Since we are using holonomic robots, we do not consider the robot orientation in the estimation process, and we assume the availability of this information with low uncertainty, such as those provided by a compass. Also, we assume that all sensor measurements are subjected to white Gaussian noise, but communication is performed without errors.

A. Initial Swarm Formation

In this work, we consider a swarm of heterogeneous robots, in which a leader has an enhanced position estimate through the use of a Global Positioning Systems (GPS). In this first step, the robots use global information (position of all other robots) to form a cohesive group. During this formation, the leader is positioned near the center of the swarm. After that, only local information is necessary to the group to perform the navigation.

Having a set of fully actuated mobile robots with dynamics given by

$$\dot{\mathbf{p}}_i = \mathbf{v}_i \text{ and } \dot{\mathbf{v}}_i = \mathbf{u}_i, \tag{1}$$

in which $\mathbf{p}_i \in \mathbb{R}^2$, $\mathbf{v}_i \in \mathbb{R}^2$, and $\mathbf{u}_i \in \mathbb{R}^2$ are the position, velocity, and control input of robot i ($\mathcal{R}_i$), respectively. The partition $\tau = \{\tau_1, \tau_2\}$, with each $\tau_k \subset \boldsymbol{\mathcal{R}} = \{\mathcal{R}_1, \mathcal{R}_2, \ldots, \mathcal{R}_\eta\}$ containing all agents of type k, models the heterogeneity of the system. We assume that $\forall j, l : j \neq l \rightarrow \tau_j \cap \tau_l = \emptyset$, i.e., each robot is uniquely assigned to a single type. In this work we consider unbalanced partitions, in which τ_1 has only one member (the leader) while τ_2 has the other robots.

The control law used in this work was proposed in [14] as follows:

$$\mathbf{u}_i = -\sum_{j \neq i} \nabla_{\mathbf{p}_i} U_{ij}(\|\mathbf{p}_{ij}\|) - \sum_{j \neq i} (\mathbf{v}_i - \mathbf{v}_j), \tag{2}$$

in which $U_{ij}(\|\mathbf{p}_{ij}\|)$ is an artificial potential function that rules the interaction between agents i and j, $\|\mathbf{p}_{ij}\|$ is the Euclidean norm of the vector $\mathbf{p}_i - \mathbf{p}_j$, and $\nabla_{\mathbf{p}_i}$ is the gradient with respect to the coordinates of agent i. The first term represents the interactions of robot i with all other agents, and the second term forces robots to match their velocities. This control law causes a group of robots to asymptotically flock as well aggregate or segregate, according to the d_{ij} parameter values (more details in the following).

The artificial potential field U_{ij} is a function of the relative distance between a pair of agents, i.e.,

$$U_{ij}(\|\mathbf{p}_{ij}\|) = \alpha \left(\frac{1}{2}(\|\mathbf{p}_{ij}\| - d_{ij})^2 + \ln \|\mathbf{p}_{ij}\| + \frac{d_{ij}}{\|\mathbf{p}_{ij}\|} \right), \tag{3}$$

in which $\alpha \in \mathbb{R}^+$ is a scalar control gain, and d_{ij} is a positive parameter that controls the group behavior.

The behavior of the group can be changed when we suitably choose the d_{ij} parameter. This parameter is a way of implementing the *differential potential* concept [15], which states that pairs of agents experience different magnitudes of potential when they have different types. So, as described in [14], the parameter d_{ij} of (3) is based on the local type partition ${}^i\tau$, such that:

$$d_{ij}({}^i\tau) = \begin{cases} d_{AA}, & \text{if } i \in \tau_k \text{ and } j \in \tau_k \\ d_{AB}, & \text{if } i \in \tau_k \text{ and } j \notin \tau_k \end{cases}. \tag{4}$$

Equation (4) asserts that interactions among similar and dissimilar types of robots are ruled by d_{AA} and d_{AB}, respectively, and by construction these constants relate to the pairwise inter-agent distance at the stable state.

As described in [14], a *segregative* behavior can be achieved when the parameters assume values considering the following rule:

$$0 < d_{AA} < d_{AB}. \tag{5}$$

On the other hand, it is shown in [16] that it is possible to achieve an *aggregative* behavior by defining the d_{ij} parameter as follows:

$$0 < d_{AB} < d_{AA} = d_{BB}. \tag{6}$$

These two rules, expressed by (5) and (6), can be combined in order to position an entire group inside another. So, we compound the aggregative and segregative behaviors by adjusting the d_{ij} parameter in such a way that

$$0 < d_{AA} < d_{AB} < d_{BB}. \tag{7}$$

In this work, we use (7) to automatically position the leader near the center of the group. As said earlier, in this work we use only two groups and the first one has only one member (the leader). Because of this, the parameter related to this group (d_{AA}) is not important and we use only the parameter associated to the second group (d_{BB}). So, for this work, the relevant part of (7) is:

$$0 < d_{AB} < d_{BB}. \tag{8}$$

Figure 1 shows a sequence of pictures that illustrates the motion of a robot (the blue one) towards the center of the other group (red one) by applying the rules described in (8). As can be seen, the group forms a circular organization around the center. In this organization, the distances between robots that are farther from the center are greater then the distances between robots that are nearer to the center. This unbalanced formation can degrade the relative measurements taken by robots that are located farther from the center more strongly than the robots that are nearer. To deal with this, we slightly changed the control law in such a way that a robot is influenced only by its neighbors:

$$\mathbf{u}_i = -\sum_{\substack{j \neq i \\ j \in \mathcal{N}_i}} \nabla_{\mathbf{p}_i} U_{ij}(\|\mathbf{p}_{ij}\|) - \sum_{\substack{j \neq i \\ j \in \mathcal{N}_i}} (\mathbf{v}_i - \mathbf{v}_j), \tag{9}$$

in which $\mathcal{N}_i$ represents the group of robots that are accessible by robot i.

A neighborhood $\mathcal{N}_i$ consists of a circular region of radius ε around the current position of robot i. Thus, we can define

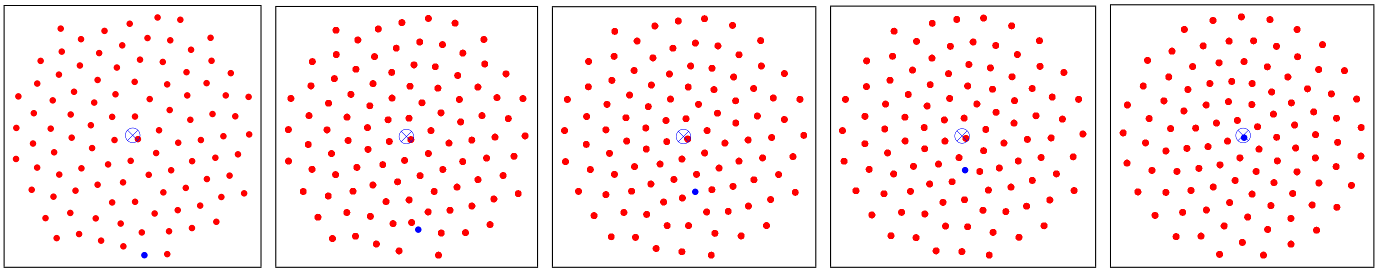


Figure 1. Snapshots of the motion of the leader (blue circle) towards the center of the swarm (marked as a blue target). In the sequence, the first and fifth images represent the initial and final configuration, respectively.

$\boldsymbol{\mathcal{N}} = \{\mathcal{N}_1, \mathcal{N}_2, \ldots, \mathcal{N}_\eta\}$ as the set of calculated neighborhoods, all with the same radius. We assume that robot i can exchange information and measure relative range ρ and bearing ϕ of all robots inside its neighborhood $\mathcal{N}_i$. Moreover, it is assumed that robots inside a neighborhood $\mathcal{N}_i$ can uniquely be identified by the exteroceptive sensor of robot i.

By using (9), the circular formation is made local and all robots present similar distances between each other. Although the artificial potential field (3) remains the same, the parameters d_{ij} are modified to achieve the aggregative behavior so that

$$0 < d_{AB} = d_{AA}. \tag{10}$$

At the first phase of the group formation, the robots use the global control law (2). In that phase, a robot needs to calculate the distances to all robots. By doing this, the leader can be automatically positioned near the center of the group by applying the rule (8). However, it is necessary to define a condition for all robots to change their control law from (2) to (9). In this work, being the leader responsible for doing this, it sends a broadcast message for all robots after reaching a position near the center.

B. Swarm Motion Strategy

In this work, after the swarm reaches an initial formation by using the steps described in the previous section, some virtual targets are defined in the environment to be used by the leader to guide the whole group. So, the leader guides the group through the environment by using an attractive potential to these targets. The control law (9) used in the previous stage by the robots is slightly changed to incorporate the attractive potential to a virtual target as:

$$\mathbf{u}_i = -\sum_{\substack{j\neq i\\ j\in\mathcal{N}_i}} \nabla_{\mathbf{p}_i} U_{ij}(\|\mathbf{p}_{ij}\|) - \sum_{\substack{j\neq i\\ j\in\mathcal{N}_i}} (\mathbf{v}_i - \mathbf{v}_j) - \delta_{il} \nabla_{\mathbf{p}_i} U_{it}(\|\mathbf{p}_{it}\|), \tag{11}$$

where $U_{it}(\mathbf{p}_i)$ is the artificial attractive potential function that guides the leader towards the target, $\mathbf{p}_t$ is the location of the target, $\|\mathbf{p}_{it}\|$ is the Euclidean distance norm of the vector $\mathbf{p}_i - \mathbf{p}_t$, and $\nabla_{\mathbf{p}_i}$ is the gradient with respect to the coordinates of the agent i. The parameter δ_{il} is the Kronecker delta function described by

$$\delta_{il} = \begin{cases} 1, & \text{if } i = l \\ 0, & \text{if } i \neq l, \end{cases}$$

where l represents the index of the leader robot.

The artificial attractive potential field U_{it} is a function of the relative distance between the leader and the target

$$U_{it}(\|\mathbf{p}_{it}\|) = \frac{1}{2}\beta\|\mathbf{p}_{it}\|^2, \tag{12}$$

in which β is a gain used to control the intensity of the potential. The adequate adjustment of this gain allows the group to maintain the formation during the motion. Despite the assumption of the knowledge of the target locations by the leader, the whole motion is performed using local sensing.

As can be seen, for a robot that is not a leader the equations (9) and (11) are the same. Then, only the leader needs to change its control law to put the group in motion, which can be done in a decentralized way. As said earlier, the control law (2) applied to swarms generates a formation that is not uniform: robots located near the center of the group are closer to its neighbors than robots that are farther from the center. Consequently, the leader is not equidistant of their immediate neighbors of d_{AB}. By using this fact, the leader puts the group in motion when its distance to each immediate neighbor has converged to values close to d_{AB}.

C. Cooperative Swarm Localization

Consider a scenario where a swarm $\boldsymbol{\mathcal{R}}$ of η holonomic robots must navigate in a static 2D environment. In the time-step k, the vectors $\mathbf{p}_i^k = [x_i^k \ y_i^k]^\top$, $\mathbf{v}_i^k = [vx_i^k \ vy_i^k]^\top$ and $\mathbf{u}_i^k = [ax_i^k \ ay_i^k]^\top$ represent the true position of the i-th robot ($\mathcal{R}_i$) in a common global frame W, the velocities, and the control action, respectively.

The state $\mathbf{x}_i^k$ of $\mathcal{R}_i$ at time-step k is defined by its position, i.e., $\mathbf{x}_i^k = \mathbf{p}_i^k$. The discrete-time motion model of robot i is expressed by:

$$\begin{aligned} \mathbf{v}_i^{k+1} &= f(\mathbf{v}_i^k, \mathbf{u}_i^k), \qquad i = 1, \ldots, \eta \\ &= \mathbf{v}_i^k + \mathbf{u}_i^k \Delta k, \\ \mathbf{x}_i^{k+1} &= f(\mathbf{x}_i^k, \mathbf{v}_i^{k+1}), \\ &= \mathbf{x}_i^k + \mathbf{v}_i^{k+1} \Delta k. \end{aligned} \tag{13}$$

The true measurements (range and bearing) taken by $\mathcal{R}_i$ of $\mathcal{R}_j$ at time-step k are respectively denoted by $\rho_{i,j}^k$ and $\phi_{i,j}^k$ [1] $(i, j = 1, \ldots, \eta, i \neq j, j \in \mathcal{N}_i)$. Thus, the true range and

[1]The notation $*_{i,j}^k$ is used to express that a certain value related to $\mathcal{R}_i$ was obtained using information or measurements from $\mathcal{R}_j$ at time-step k.

bearing taken at time-step k by robot i of robot j, is given by $h(\mathbf{x}_i^k, \mathbf{x}_j^k)$, where

$$h(\mathbf{x}_i^k, \mathbf{x}_j^k) = \begin{bmatrix} \rho_{i,j}^k \\ \phi_{i,j}^k \end{bmatrix} = \begin{bmatrix} \sqrt{(x_j^k - x_i^k)^2 + (y_j^k - y_i^k)^2} \\ \text{atan2}(y_j^k - y_i^k, x_j^k - x_i^k) \end{bmatrix}. \quad (14)$$

The measurement model $\mathbf{z}_{i,j}^{k+1} = [\hat{\rho}_{i,j}^{k+1} \; \hat{\phi}_{i,j}^{k+1}]$ at time-step $k+1$, when $\mathcal{R}_i$ gets a relative position measurement of $\mathcal{R}_j$, is given by

$$\mathbf{z}_{i,j}^{k+1} = h(\mathbf{x}_i^{k+1}, \mathbf{x}_j^{k+1}) + \mathbf{n}_{i,j}^{k+1}, \quad (15)$$

where $\mathbf{n}_{i,j}^{k+1}$ is the zero-mean white Gaussian measurement noise with covariance $\mathbf{R}_{i,j}^{k+1}$ added to the true relative measurements given by $h(\mathbf{x}_i^{k+1}, \mathbf{x}_j^{k+1})$.

In this work, the swarm motion is coordinated such that the group moves as a unit (see Section III-B). The robots use only local information and, individually, each robot i maintains only its own state estimate $\hat{\mathbf{x}}_i^k$ and covariance $\mathbf{P}_i^k$. This permit us to deal with the costs of processing and communication when cooperatively localizing a large group of robots.

The cooperative localization is performed as follows. Robots with uncertainty level lower than a prespecified value act as landmarks and broadcast messages composed of their state and covariance to its neighbors. After reaching the prespecified value of uncertainty, a robot is not considered a landmark anymore and stop to broadcast messages. During the motion, after a robot has accumulated a given level of uncertainty, it tries to localize itself by processing the relative range and bearing measurements $\mathbf{z}_{i,j}$ together with the information received from its landmark neighbors. Because of the uniform formation generated by (9), the landmark selection is basically done based only upon the uncertainty level of the information received. This way, the neighbor with the lowest uncertainty level is selected, and its data are fused with the robot's predicted state $\hat{\mathbf{x}}_i^{k+1|k}$ and covariance $\mathbf{P}_i^{k+1|k}$ estimates[2] to generate the new state $\hat{\mathbf{x}}_i^{k+1|k+1}$ and covariance $\mathbf{P}_i^{k+1|k+1}$ estimates. The mathematical details of this procedure is presented as follows.

The propagation of a robot's state is performed by using the discrete-time motion model described in (13), such as:

$$\hat{\mathbf{x}}_i^{k+1|k} = f(\hat{\mathbf{x}}_i^{k|k}, \hat{\mathbf{v}}_i^{k+1}), \qquad i = 1, \ldots, \eta, \quad (16)$$

which consists of a function f that considers the previous state $\hat{\mathbf{x}}_i^k$ and the velocity $\hat{\mathbf{v}}_i^{k+1} = \hat{\mathbf{v}}_i^k + \hat{\mathbf{u}}_i^k \Delta k$. The input $\hat{\mathbf{u}}_i^k = \mathbf{u}_i^k + \mathbf{w}_i^k = [\widehat{ax}_i^k \; \widehat{ay}_i^k]^\top$ is basically the commanded accelerations $\mathbf{u}_i^k$ augmented with additive zero-mean white Gaussian noise $\mathbf{w}_i^k$, with covariance $\mathbf{Q}_i^k$. During the motion, each robot individually evolves this model with time-steps of length Δk.

Using an EKF [7], the respective covariance propagation for $\mathcal{R}_i$ is given by:

$$\mathbf{P}_i^{k+1|k} = \Phi_i^k \mathbf{P}_i^{k|k} (\Phi_i^k)^\top + \mathbf{G}_i^k \mathbf{Q}_i^k (\mathbf{G}_i^k)^\top, \quad (17)$$

[2]Similar notation to [9] is used here: $\hat{\mathbf{y}}^{l|m}$ denotes the estimate of the random variable $\mathbf{y}$ at time-step l, given the measurements up to time-step m.

where Φ_i^k is a 2×2 identity matrix ($\mathbf{I}_2$) and $\mathbf{G}_i^k$ is this same matrix multiplied by the time-step Δk.

When $\mathcal{R}_i$ is trying to localize itself, it fuses the best data among those received by its neighbors with the respective relative measurement $\mathbf{z}_{i,j}^{k+1}$ taken. This way, $\mathcal{R}_i$ can generate an estimate of its own state as if such estimate had been calculated by the robot $\mathcal{R}_j$, as illustrated in the following equation:

$$\hat{\mathbf{x}}_{i,j}^{k+1} = \hat{\mathbf{x}}_j^{k+1|k+1} - g(\mathbf{z}_{i,j}^{k+1}), \quad (18)$$

where

$$g(\mathbf{z}_{i,j}^{k+1}) = \begin{bmatrix} \hat{\rho}_{i,j}^{k+1} \cos(\hat{\phi}_{i,j}^{k+1}) \\ \hat{\rho}_{i,j}^{k+1} \sin(\hat{\phi}_{i,j}^{k+1}) \end{bmatrix}.$$

The approach used here is similar to [17] which is based on [18]. The uncertainty $\mathbf{R}_{i,j}^{k+1}$ tied to the measurement $\mathbf{z}_{i,j}^{k+1}$ can be converted to the common frame W by the means of the jacobian $\mathbf{J}_{i,j}^{k+1}$, as follows:

$$\begin{aligned} \mathbf{J}_{i,j}^{k+1} &= \nabla_{\mathbf{x}^k} g(\mathbf{z}_{i,j}^{k+1})\big|_{\mathbf{x}_i^k = \hat{\mathbf{x}}_i^{k+1|k}, \mathbf{x}_{i,j}^k = \hat{\mathbf{x}}_{i,j}^{k+1}} \\ &= \begin{bmatrix} \cos(\hat{\phi}_{i,j}^{k+1}) & -\hat{\rho}_{i,j}^{k+1} \sin(\hat{\phi}_{i,j}^{k+1}) \\ \sin(\hat{\phi}_{i,j}^{k+1}) & \hat{\rho}_{i,j}^{k+1} \cos(\hat{\phi}_{i,j}^{k+1}) \end{bmatrix}. \end{aligned} \quad (19)$$

The jacobian $\mathbf{J}$ relates the deviation of the original $[\Delta\hat{\rho} \; \Delta\hat{\phi}]^\top$ and the transformed $[\Delta\hat{x} \; \Delta\hat{y}]^\top$ variables, which represent the distances from $\mathcal{R}_i$ and $\mathcal{R}_j$ in x and y coordinates, respectively, calculated as:

$$\begin{bmatrix} \Delta\hat{x} \\ \Delta\hat{y} \end{bmatrix} = \mathbf{J} \begin{bmatrix} \Delta\hat{\rho} \\ \Delta\hat{\phi} \end{bmatrix}. \quad (20)$$

The covariance of the measurement $\mathbf{z}$ in the common frame is defined by multiplying both sides of Equation (20) by their respective transposes and taking the expectation of the result. This transformation represents an adequate linear approximation when the variables are represented by Gaussians with small variances, as stated in [18]. The uncertainty $\mathbf{P}_{i,j}^{k+1}$ related to the $\hat{\mathbf{x}}_{i,j}^{k+1}$ estimate is generated by the combination of the covariance matrices:

$$\mathbf{P}_{i,j}^{k+1} = \mathbf{P}_i^{k+1|k} + \mathbf{J}_{i,j}^{k+1} \mathbf{R}_{i,j}^{k+1} (\mathbf{J}_{i,j}^{k+1})^\top. \quad (21)$$

In the update step of the EKF, the estimates $\hat{\mathbf{x}}_i^{k+1|k}$ and $\hat{\mathbf{x}}_{i,j}^{k+1}$ are combined to generate a new state $\hat{\mathbf{x}}_i^{k+1|k+1}$ and covariance $\mathbf{P}_i^{k+1|k+1}$. These represent the actual belief of the robot, and are used in the next prediction phase of the filter.

IV. Simulations

We performed a series of experiments to show the effectiveness of the proposed methodology. Specifically, we analyzed the quality of the localization obtained using the Cooperative Localization and the cohesive navigation. The experiments were executed considering a swarm with 100 holonomic small robots. The group navigates in an obstacle-free static environment of approximately two hundred square meters ($15\,\text{m} \times 15\,\text{m}$). Swarm motion is directed by a series of waypoints, which define virtual targets to be reached by the group. The location of these targets is used in (11) to calculate the last term of the equation, which generates a control action that guides the leader, and consequently the others, to specific regions of the environment. The controller generated acceleration is subjected to additive zero-mean Gaussian noise with a standard deviation of 10% of the true value. Each simulation takes approximately 56300 time-steps, where $\Delta k = 10\,\text{ms}$ is the duration of each time-step. The total length of the traveled distance is about $35.5\,\text{m}$.

Following the methodology, the robots are initially assembled together, and the leader is located in a random position. By using (2), the leader is guided towards the center of the group (see Figure 1). In this phase, the d_{AB} parameter was set to 0.5, while the d_{AA} was set to 1. The localization algorithm (Section III-C) uses relative measurements between pairs of robots in the group. Because of this, the non uniformity of the formation generated by (2) faster degrades the localization quality of the robots that are farther from the center of the swarm. To deal with this, the *local* control law (9) is applied to generate a more uniform formation, while the usage of (11) allows the robots to move by using only local information. In these control laws, the parameters used by the potential function (3) were set to: $d_{AA} = d_{AB} = 0.2$. The radius ε was defined as $0.266\,\text{m}$ $(1.33 \times d_{AB})$, which makes each robot to have access only to robots that are located next to it.

Figure 2 presents an example of a path executed by the swarm in the environment. The large gray circles depict the regions that the swarm needs to visit, while their centers define the targets (waypoints). The position of the robots is presented over time as *dots*. The blue one characterizes the robot leader while the red ones represent the other robots. We also highlight the robots' positions in five distinct moments during the swarm motion. It is possible to see that the swam maintains the formation during the motion.

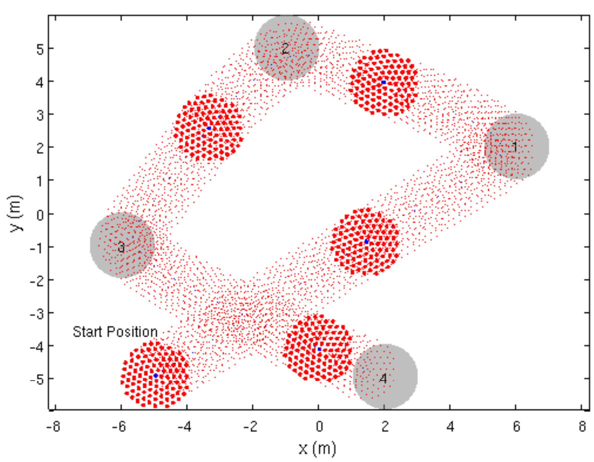


Figure 2. A path executed by the swarm. The blue color denotes the leader robot while the red one indicates the other robots.

In the following, we show the results of three types of experiments: (i) the localization is done such that each robot uses only its proprioceptive sensors (without cooperation); (ii) the cooperative localization is performed but no robot is equipped with GPS; and (iii) a leader equipped with GPS is used as the best estimate of the group and propagates this information to its neighbors. When using cooperative localization, each robot localizes itself by using only its best neighbor (lower uncertainty) as landmark. In these experiments, the range and bearing noise were defined proportional to the actual measurements and were set considering a standard deviation of 5% of the true measurements.

In this analysis, each experiment was performed 30 times and the RMSE error was calculated. The objective is to compare the localization error of robots based on the distance that they are from the leader. To help the analysis of this error, the swarm was divided in six groups of robots by using the k-means clustering method. The criterion used to assign a robot to the group i (G_i) was the actual distance of such robot to the leader. Figure 3 shows the division performed when using $k = 6$. The circles represent the robots and each color characterize a group. The leader is represented by the white color and was considered as the reference point in the clustering process.

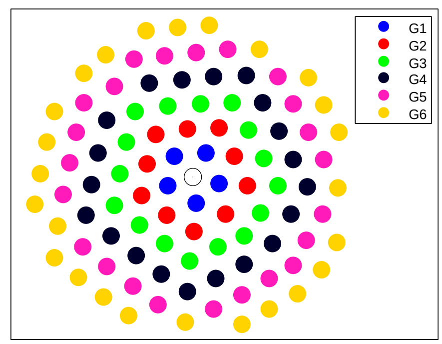


Figure 3. Clustering used for analysis purposes only. The leader (white circle) is located at the center of the group while each other robot is assigned to a group (G_1 to G_6) based on its actual distance to the leader.

The RMSE localization error was performed considering these groups. Figure 4 shows the error for the three types of experiments. It is possible to note the increasing in the localization quality when using cooperation (CL without GPS). Moreover, the usage of only one robot equipped with GPS increases significantly the quality of the individual localization (CL with GPS (leader)). Despite not shown in this figure, we also observed in the experiments that the localization error increases much faster when not using cooperation (no cooperation).

Figure 5 presents an analysis of the error related to the experiments in which the leader was equipped with GPS and made use of cooperative localization. The G_1 group was used as reference because its proximity with the leader. So, each bar shows the proportion of the group error in relation to G_1, i.e. G_i/G_1. It is possible to note the tendency of increasing in the error when a group is farther from G_1. The localization error of G_6 is almost fifty percent worst than the localization of group G_1. In Figure 4, the groups error in cooperative localization

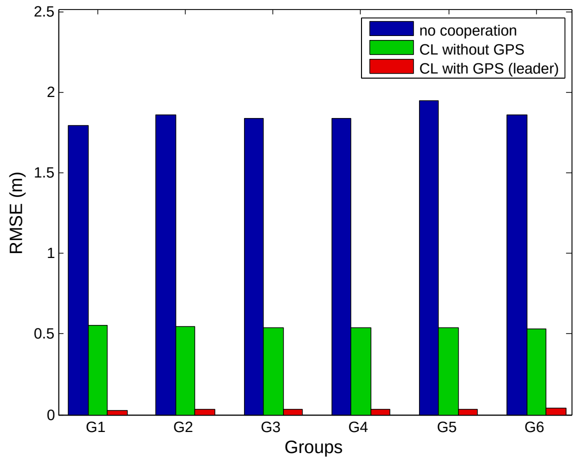


Figure 4. Localization error considering three different types of experiments.

when no robots was equipped with GPS (CL without GPS) are almost the same.

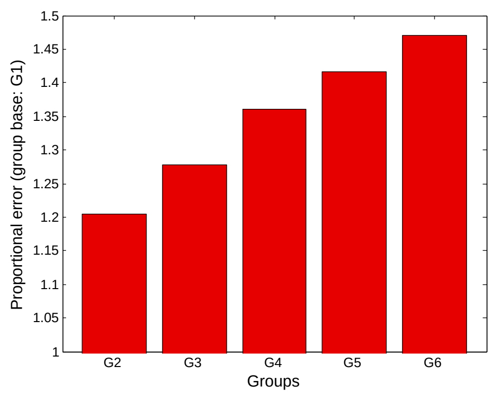


Figure 5. Cooperative localization error with a robot equipped with GPS (CL with GPS (leader)). Each group G_i is defined by the distance of its robots to the leader (less i value means closer to the leader).

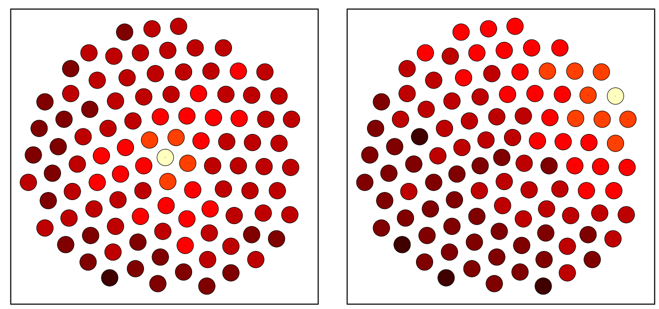
Table I summarizes the cooperation performed during cooperative localization (CL with GPS (leader)). It shows the proportions (averaged over the 30 runs) in which a group made use of the others. For example, we can see that robots in Group G2 used robots in G1 in 53% of the localization updates, G2 in 26% and G3 in 21%. It is important to note that robots that are part of G1 corrected their position using the leader, which is illustrated by the value 0 in all groups. As said before, the radius ε was defined in such a way that each robot can only exchange information with robots that are adjacent to it. Consequently, a group is associated only with groups that are near it. Moreover, we can observe that each group, most of the time, uses its neighboring group that is nearer to the leader.

The importance of the leader positioning is shown in Figure 6. As said earlier, robots that are farther from the leader accumulate more error. So, when only one robot with GPS is used in the swarm, its best location is near the center of the group. In this figure, the light circle depicts the location of the leader, while the darker ones represent the higher errors.

Table I
INTERGROUP COOPERATION

		Groups					
		G1	G2	G3	G4	G5	G6
Groups	G1	0	0	0	0	0	0
	G2	0.53	0.26	0.21	0	0	0
	G3	0	0.43	0.32	0.25	0	0
	G4	0	0	0.42	0.28	0.28	0.02
	G5	0	0	0.02	0.43	0.25	0.30
	G6	0	0	0	0.08	0.63	0.29

In Figure 6(b), in which the leader is not at the center, it is possible to note that robots located in the opposite side of the group have higher error than robots located in the vicinity of the leader.



(a) (b)

Figure 6. Analysis of the influence of the leader position in the overall localization quality. The light circle represents the leader location: (a) at center; (b) in a frontier of the group. The colors depict the intensity of the localization error. Higher errors are represented by darker circles.

To help the analysis of the localization deterioration related to the distance of a robot to the leader, we changed the number of clusters used when the leader is positioned in the frontier of the group (white circle). The position errors of the robots located in one of the fifteen clusters (see Figure 7) were averaged and associated with the distance of the respective cluster to the leader, which is presented in Figure 8. As can be seen, the tendency of increasing discussed earlier is maintained. Figure 8 suggests that in larger swarms only one robot with absolute positioning may not be enough to maintain all members of the group with good level of localization and that the position of this robot is a key factor.

V. Conclusion and Future Work

In this paper we dealt with the cooperative localization problem applied to a swarm of robots. Considering the sensory and computational limitations of the robots usually used in swarms, we proposed the use of a special type of agent (leader) equipped with a GPS, with the objective of providing good position estimates to the rest of the group. The leader is placed among the swarm, enabling the robots to cooperatively localize themselves using local information through an approximate decentralized algorithm.

Despite the fact that the use of cooperation improves the localization of the group, the continuous motion based solely on proprioceptive sensors degrades the localization quality

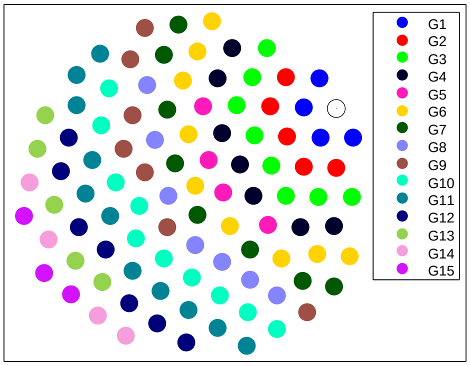


Figure 7. Increased number of clusters used to analyze the impact of the distance to the leader (white circle) for the localization.

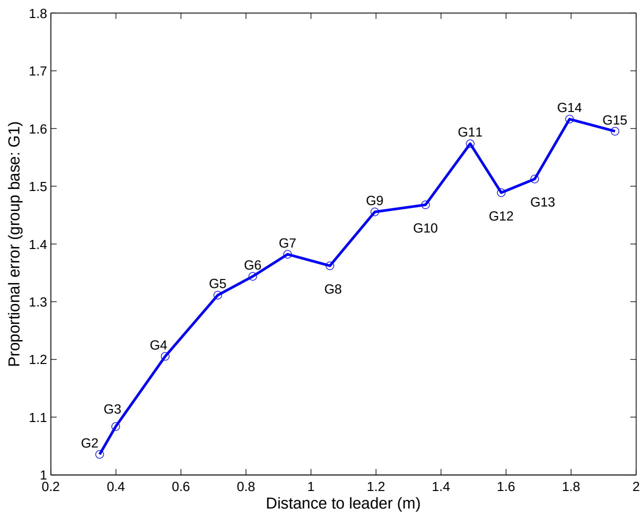


Figure 8. Localization error related to the distance of a robot in a certain group to the leader.

with the distance traveled. Heterogeneity, in the context of cooperative localization, is an option to deal with this problem and we show here that even using only a single robot with special localization capabilities in a swarm can further improve the localization. However, to take the maximum advantage of this robot, it is important to choose a good position to place it. In this work, we presented a way of automatically positioning this robot in the group and, moreover, to maintain the group formation.

Here we have obtained evidences that only one robot with improved sensory capabilities may not be enough in larger groups. Therefore, future directions include the extension of the proposed methodology to deal with more than one leader robot equipped with sensors that may increase the quality in the position estimate. Also, we intend to study strategies to automatically position these leaders in the group. The increase in the number of such robots may also benefit the robustness of the approach.

Acknowledgments

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