1) For each of the problems below define the sample space S. State whether S is finite, countably infinite, or uncountably infinite. If S is finite, calculate |S|, the number of elements in the set S.
   a. Toss a coin 5 times and count the number of heads.
   b. Toss a coin until the first “heads” shows up and count the number of tosses of “tails that occur before this happens.
   c. Roll 4 dice and compute their sum.
   d. Roll 2 dice and compute their product.
   e. Measure the time (rounded to integer minutes) that it takes for a disk to fail from the time it is installed.
   f. Measure the number of unique customers (rounded to integer millions) with US addresses that visit the Amazon.com Web site in any given 24-hour period. “Unique” here means that if a person visits the site more than once during the time period they are only counted once and not multiple times.

2) Two chess players A and B decide to play each other in 5 online chess games. Let \( A_k \); \( B_k \); \( D_k \) represent the events that A wins game k, B wins game k, or D wins game k, respectively, where \( k = 1; \ldots ; 5 \). Describe the following events in terms of \( A_k \), \( B_k \), and \( D_k \),. For example, the event that A wins game 1 and B wins game 2 would be described as \( A_1 \cap B_2 \).
   a. A wins at least 1 of the first 3 games.
   b. B does not win any of the first 3 games.
   c. None of the 5 games end in a draw.
   d. There is 1 draw (and no more) over the 5 games.
   e. A wins 2 games in a row at least once during the series of 5 games. If A wins more than 3 or more games in a row, this counts as winning 2 games in a row, so include this in your definition of the event.

3) Consider a communications network where each packet travels over 2 different sub-networks to get from one computer to another. In each sub-network the packet can take 1, 2, 3, 4, or 5 hops, where each of the 5 possibilities is equally likely. The number of hops in one network does not depend in any way on the number of hops taken in the other network (i.e., they are independent). Let X be the random variable corresponding to the total number of hops taken by a packet.
   a. What is the probability mass function (pmf) for X?
   b. Say each hop causes a delay of 1 millisecond. What is the expected delay for a packet in traversing the two sub-networks? (i.e., what is \( E[X] \)?)
4) Say you have chosen a 6-character password for a particular computer account, where English letters or the digits 0 through 9 are allowed in each position, and that is not case-sensitive (upper and lower letters are treated the same way), e.g., ez1ad2.

a. If someone guesses randomly at your password, what is the probability that they guess it correctly on their first guess?
b. What if they write a computer program that randomly selects 1 million (different) passwords and tries them all -- what is the probability that none of these passwords match the correct password?
c. Say that your password really is ez1ad2 and that the person guessing knows (somehow) that your password has vowels in the first and 4th positions, non-vowel letters in the 2nd and 5th positions, and digits in the 3rd and 6th positions.