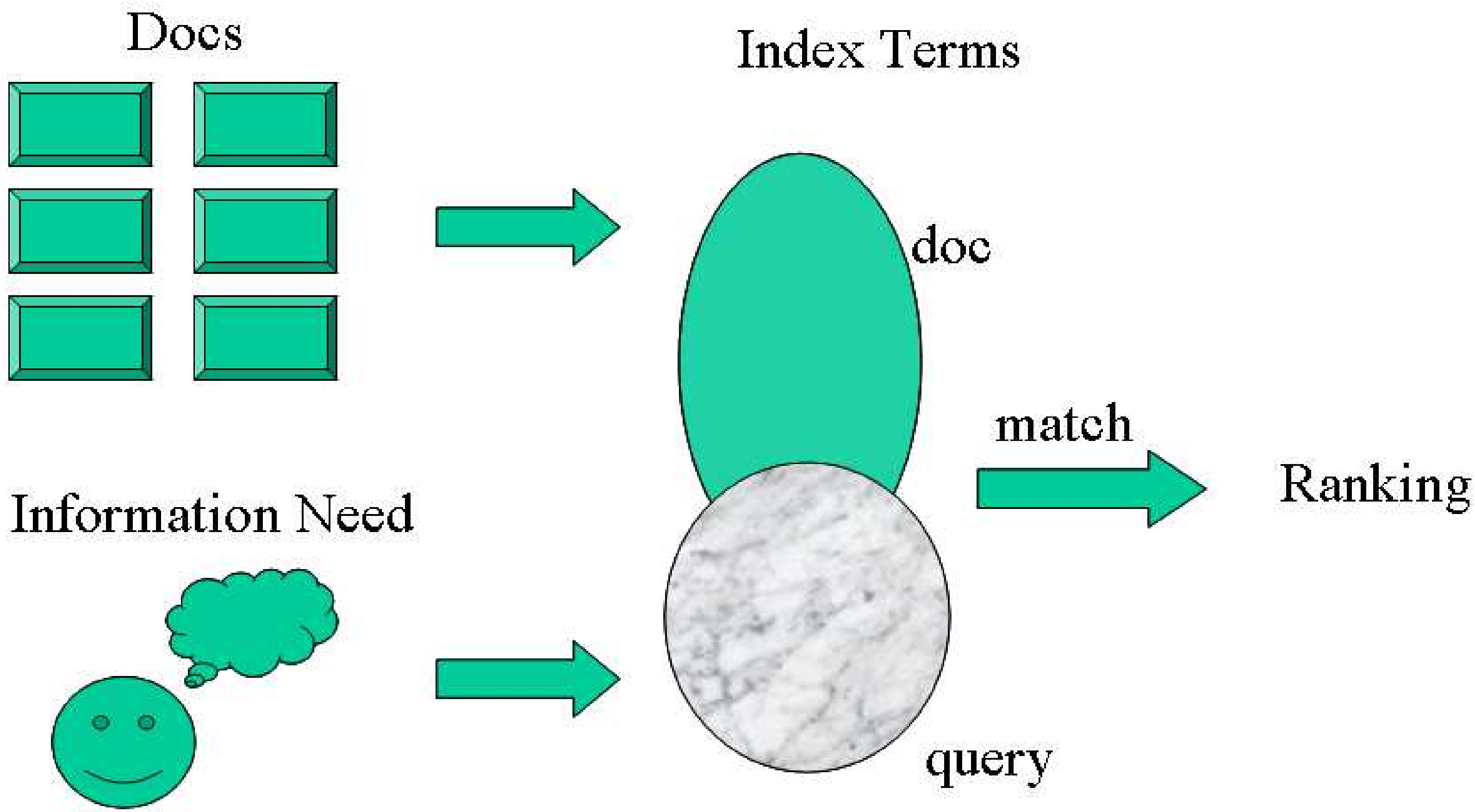
Chapter 2

Modeling

Introduction

- IR systems usually adopt index terms to process queries
- Index term:
  - a keyword or group of selected words
  - any word (more general)
- Stemming might be used:
  - connect: connecting, connection, connections
- An inverted file is built for the chosen index terms

Introduction



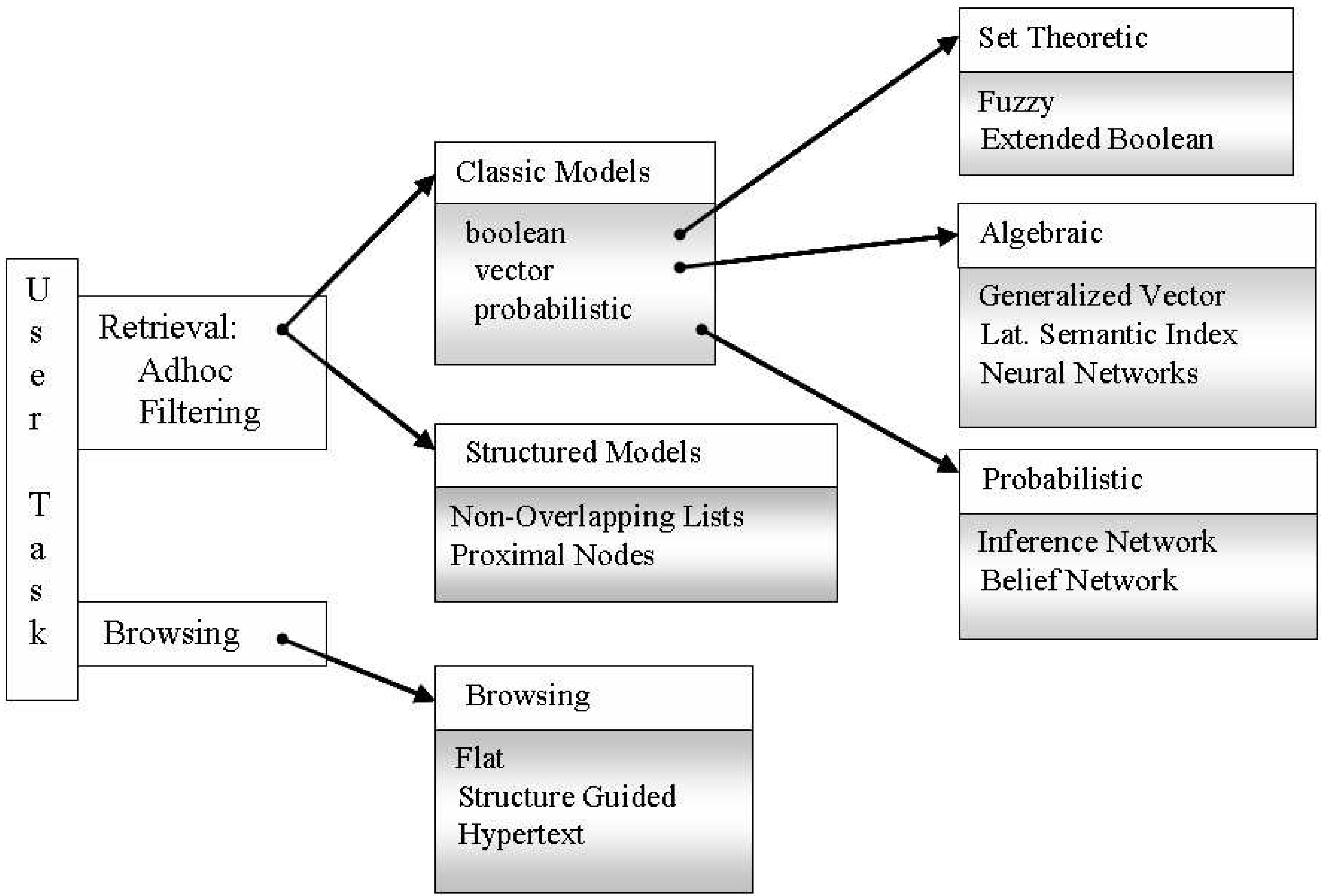
Introduction

- Matching at index term level is quite imprecise
- No surprise that users get frequently unsatisfied
- Since most users have no training in query formation, problem is even worst
- Frequent dissatisfaction of Web users
- Issue of deciding relevance is critical for IR systems: ranking

Introduction

- A ranking is an ordering of the documents retrieved that (hopefully) reflects the relevance of the documents to the user query
- A ranking is based on fundamental premisses regarding the notion of relevance, such as:
  - common sets of index terms
  - sharing of weighted terms
  - likelihood of relevance
- Each set of premisses leads to a distinct IR model

IR Models



User Task
Retrieval:
Adhoc
Filtering
Browsing
Classic Models
boolean
vector
probabilistic
Structured Models
Non-Overlapping Lists
Proximal Nodes
Browsing
Flat
Structure Guided
Hypertext
Set Theoretic
Fuzzy
Extended Boolean
Algebraic
Generalized Vector
Lat. Semantic Index
Neural Networks
Probabilistic
Inference Network
Belief Network

IR Models

- The IR model, the logical view of the docs, and the retrieval task are distinct aspects of the system

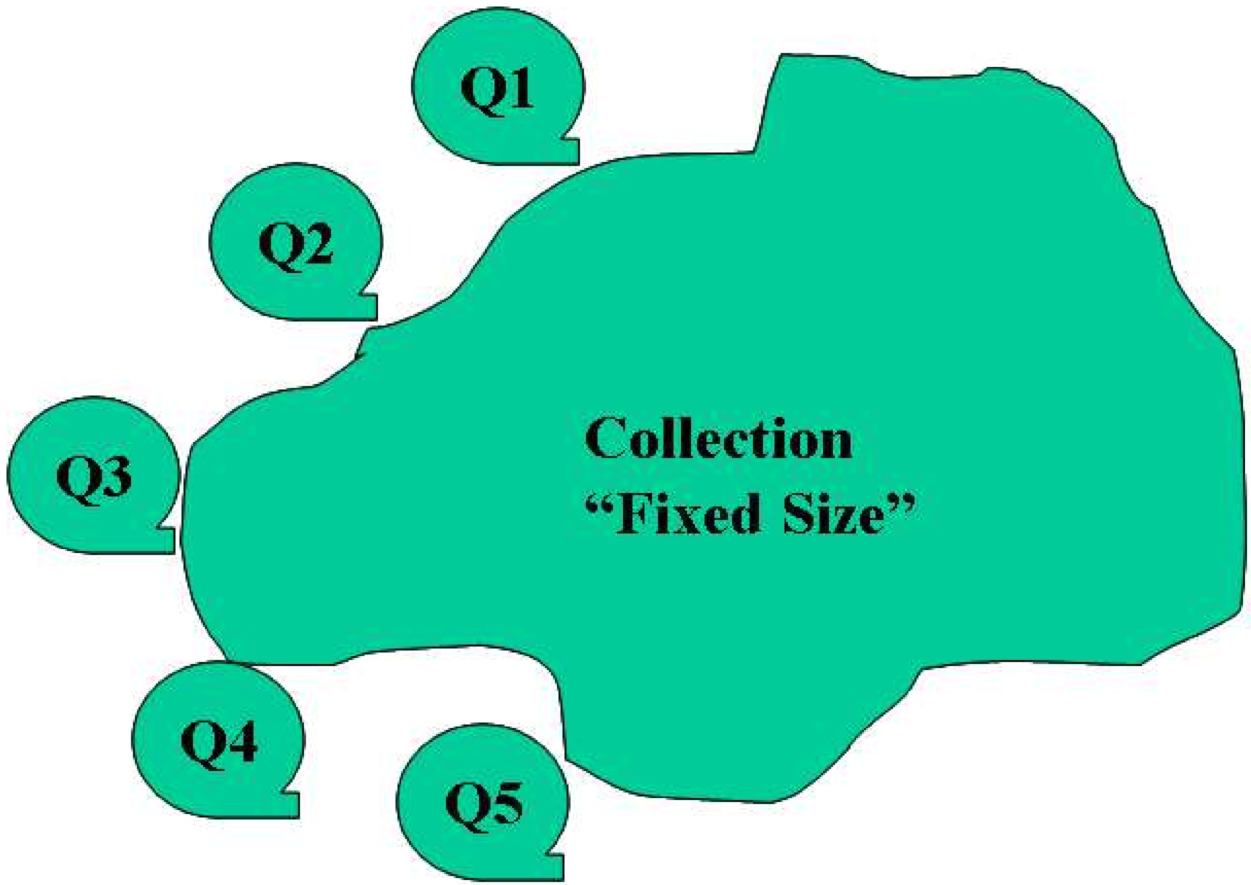
LOGICAL VIEW OF DOCUMENTS

USER TASK

	Index Terms	**Full Text**	**Full Text + Structure**
Retrieval	Classic Set Theoretic Algebraic Probabilistic	Classic Set Theoretic Algebraic Probabilistic	Structured
Browsing	Flat	Flat Hypertext	Structure Guided Hypertext

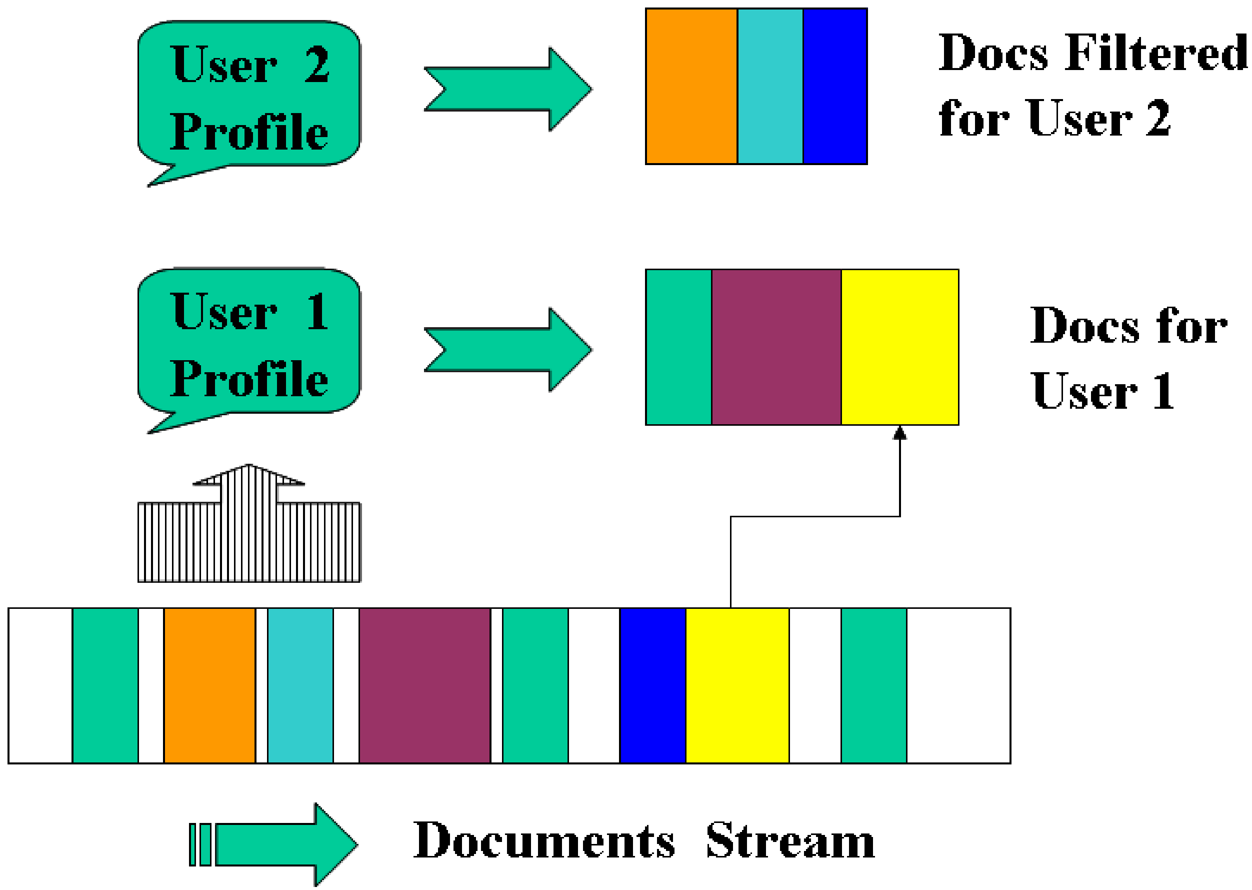
Retrieval: Ad Hoc x Filtering

- Ad Hoc Retrieval:



Retrieval: Ad Hoc x Filtering

- Filtering



Classic IR Models - Basic Concepts

- Each document represented by a set of representative keywords or index terms
- An index term is a document word useful for remembering the document main themes
- Usually, index terms are nouns because nouns have meaning by themselves
- However, search engines assume that all words are index terms (full text representation)

Classic IR Models - Basic Concepts

- Not all terms are equally useful for representing the document contents
  - less frequent terms allow identifying a narrower set of documents
- To quantify the importance of an index term, we associate a weight with it
- Let
  - k_i be an index term
  - d_j be a document
  - w_{ij} be a weight associated with (k_i , d_j), which quantifies the importance of k_i for describing the contents of d_j

Classic IR Models - Basic Concepts

- Let
  - k_i : i^{th} index term
  - d_j : j^{th} document
  - t : total number of terms in the vocabulary
  - $K = \{k_1, k_2, ..., k_t\}$: the set of all index terms
  - $w_{ij} \geqslant 0$: weight associated with (k_i, d_j)
    - if $w_{ij} = 0$ then term k_i does not occur within d_j
  - $\vec{d}_j = (w_{1j}, w_{2j}, ..., w_{tj})$: weighted vector associated with d_j
  - $g(\vec{d}_j)$: a reference to the weight w_{ij}

The Boolean Model

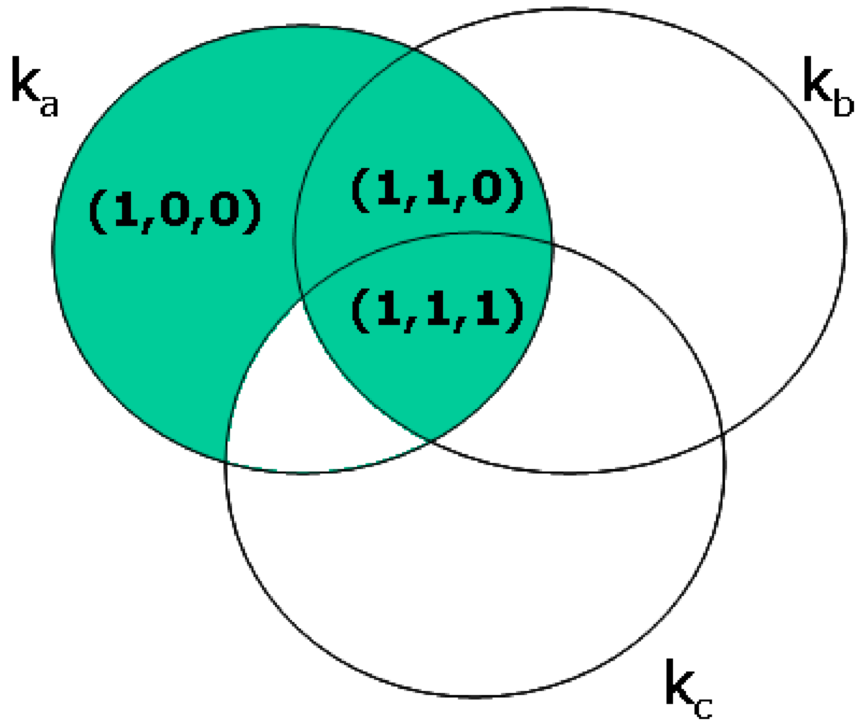
- Simple model based on set theory
- Queries specified as Boolean expressions
  - precise semantics
  - neat formalism
  - $q = k_a \wedge (k_b \vee \neg k_c)$
- Let
  - w_{iq} : weight associated with pair (k_i, q)
  - $w_{iq} \in \{0, 1\}$: terms either present or absent (Boolean)
  - $\vec{d}_q = (w_{1q}, w_{2q}, ..., w_{tq})$: weighted vector associated with q

The Boolean Model

- Let,
  - $q = k_a \wedge (k_b \vee \neg k_c)$
  - $\vec{d}_q$: weighted vector associated with q
  - $dnf(\vec{d}_q)$: distinct normal form for vector $\vec{D}_q$
- Then,
  - $dnf(\vec{d}_q) = (1,1,1) \vee (1,1,0) \vee (1,0,0)$
    - (1,1,1) : conjunctive component for (k_a, k_b, k_c)
    - (1,1,0) : conjunctive component for $(k_a, k_b, \neg k_c)$
    - (1,0,0) : conjunctive component for $(k_a, \neg k_b, \neg k_c)$
  - cc_q : a conjunctive component for $q, cc_q \in dnf(\vec{d}_q)$

The Boolean Model

- $q = k_a \wedge (k_b \vee \neg k_c)$



- $sim(q, d_j) = 1$, if $\exists cc_q | \forall k_i, g_i(\vec{d_j}) = g_i(cc_q)$
- $sim(q, d_j) = 0$, otherwise

Drawbacks of the Boolean Model

- Retrieval based on binary decision criteria with no notion of partial matching
- No ranking of the documents is provided (absence of a grading scale)
- Information need has to be translated into a Boolean expression, which most users find awkward
- The Boolean queries formulated by the users are most often too simplistic
- As a consequence, the Boolean model frequently returns either too few or too many documents in response to a user query

The Vector Model

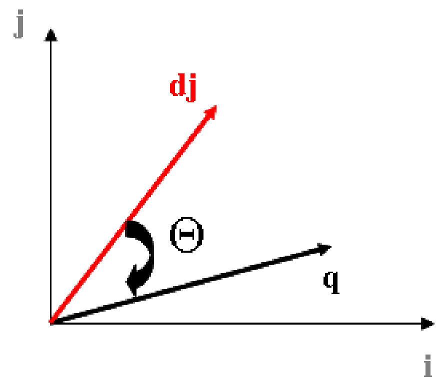
- Use of binary weights is too limiting
- Non-binary weights provide consideration for partial matches
- These term weights are used to compute a degree of similarity between a query and each document
- Ranked set of documents provides for better matching

The Vector Model

- Define:
  - $w_{ij} > 0$ whenever $k_i \in d_j$
  - $w_{iq} \geqslant 0$ associated with the pair (k_i, q)
  - $\vec{d_j} = (w_{1j}, w_{2j}, \ldots, w_{tj})$
  - $\vec{d_q} = (w_{1q}, w_{2q}, \ldots, w_{tq})$
  - To each term k_i is associated a unitary vector $\vec{i}$
  - The unitary vectors $\vec{i}$ and $\vec{j}$ are assumed to be orthonormal (i.e., index terms are assumed to occur independently within the documents)
- The t unitary vectors $\vec{i}$ form an orthonormal basis for a t-dimensional space
- In this space, queries and documents are represented as weighted vectors

The Vector Model

- Similarity



$$sim(d_j, q) = cos(\theta) = \frac{\vec{d_j} \bullet \vec{q}}{|\vec{d_j}| \times |\vec{q}|} = \frac{\sum_{i=1}^{t} w_{i,j} \times w_{i,q}}{\sqrt{\sum_{i=1}^{t} w_{i,j}^2} \times \sqrt{\sum_{j=1}^{t} w_{i,q}^2}}$$

- Since $w_{ij} > 0$ and $w_{iq} > 0$ then $0 \leqslant sim(d_j, q) \leqslant 1$
- A document is retrieved even if it matches the query terms only partially

The Vector Model

- Similarity

$$sim(d_j, q) = \frac{\sum_{i=1}^{t} w_{i,j} \times w_{i,q}}{\sqrt{\sum_{i=1}^{t} w_{i,j}^2} \times \sqrt{\sum_{j=1}^{t} w_{i,q}^2}}$$

- How to compute the weights w_{ij} and w_{iq}?
- A good weight must take into account two effects:
  - quantification of intra-document contents (similarity)
    - *tf factor*, the term frequency within a document
  - quantification of inter-documents separation (dissimilarity)
    - *idf factor*, the inverse document frequency
  - $w_{i,j} = tf_{i,j} \times idf_i$

The Vector Model

- Let, $k_i \in d_j$
  - N be the total number of docs in the collection
  - n_i be the number of docs which contain k_i
  - $freq_{i,j}$ raw frequency of k_i within d_j
- A normalized *tf factor* is given by
  - $f_{i,j} = \frac{freq_{i,j}}{max_l \quad freq_{l,j}}$
  - where the maximum is computed over all terms which occur within the document d_j
- The *idf factor* is computed as
  - $idf_i = \log \frac{N}{n_i}$
  - the log is used to make the values of *tf* and *idf* comparable. It can also be interpreted as the amount of information associated with the term k_i.

The Vector Model

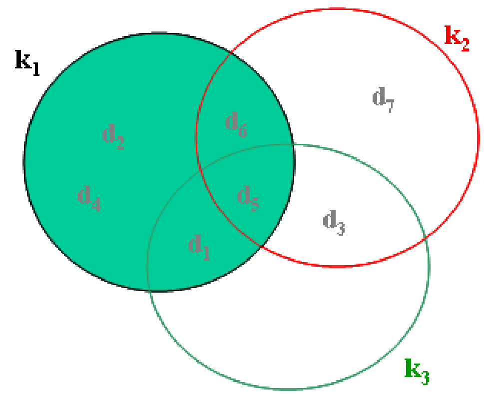
- The best term-weighting schemes use weights which are give by
  - $w_{i,j} = f_{i,j} \times \log \frac{N}{n_i}$
  - the strategy is called a tf-idf weighting scheme
- For the query term weights, a suggestion is
  - $w_{i,q} = \left(0.5 + \frac{0.5 \ freq_{i,q}}{max_l \ \ freq_{l,q}}\right) \times \log \frac{N}{n_i}$
- The vector model with *tf-idf* weights is a good ranking strategy with general collections
- The vector model is usually as good as the known ranking alternatives. It is also simple and fast to compute.

The Vector Model

- Advantages:
  - term-weighting improves quality of the answer set
  - partial matching allows retrieval of docs that approximate the query conditions
  - cosine ranking formula sorts documents according to degree of similarity to the query
- Disadvantages:
  - assumes independence of index terms (??); not clear that this is bad though

The Vector Model

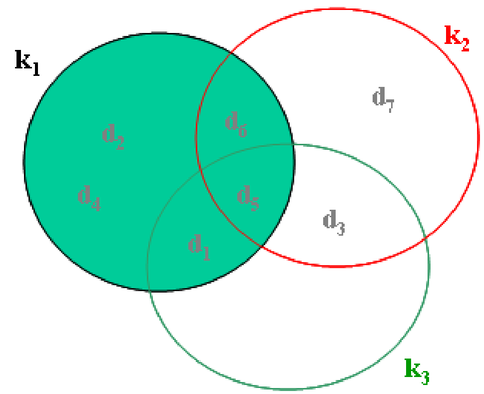
Example 1



	k_1	k_2	k_3	$q \bullet d_j$
d_1	1	0	1	**2**
d_2	1	0	0	**1**
d_3	0	1	1	**2**
d_4	1	0	0	**1**
d_5	1	1	1	**3**
d_6	1	1	0	**2**
d_7	0	1	0	**1**
q	1	1	1	

The Vector Model

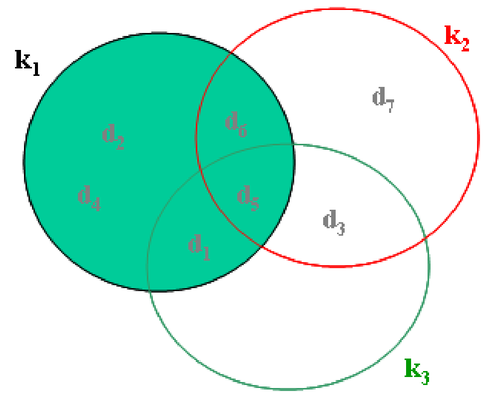
Example 2



	k_1	k_2	k_3	$q \bullet d_j$
d_1	1	0	1	4
d_2	1	0	0	1
d_3	0	1	1	5
d_4	1	0	0	1
d_5	1	1	1	6
d_6	1	1	0	3
d_7	0	1	0	2
q	1	2	3	

The Vector Model

Example 3



	k_1	k_2	k_3	$q \bullet d_j$
d_1	2	0	1	**5**
d_2	1	0	0	**1**
d_3	0	1	3	**11**
d_4	2	0	0	**2**
d_5	1	2	4	**17**
d_6	1	2	0	**5**
d_7	0	5	0	**10**
q	1	2	3	

Probabilistic Model

- Objective: to capture the IR problem using a probabilistic framework
- Given a user query, there is an ideal answer set
- Querying as specification of the properties of this ideal answer set (clustering)
- But, what are these properties?
- Guess at the beginning what they could be (i.e., guess initial description of ideal answer set)
- Improve by iteration

Probabilistic Model

- An initial set of documents is retrieved somehow
- User inspects these docs looking for the relevant ones (in truth, only top 10-20 need to be inspected)
- IR system uses this information to refine description of ideal answer set
- By repeating this process, it is expected that the description of the ideal answer set will improve
- Have always in mind the need to guess at the very beginning the description of the ideal answer set
- Description of ideal answer set is modeled in probabilistic terms

Probabilistic Ranking Principle

- Given a user query q and a document d_j, the probabilistic model tries to estimate the probability that the user will find the document d_j interesting (i.e., relevant). The model assumes that this probability of relevance depends on the query and the document representations only. Ideal answer set is referred to as R and should maximize the probability of relevance. Documents in the set R are predicted to be relevant.
- But,
  - how to compute probabilities?
  - what is the sample space?

The Ranking

- Similarity

$$sim(d_j, q) = \frac{P(R|\vec{d_j})}{P(\overline{R}|\vec{d_j})} = \frac{P(\vec{d_j}|R) \times P(R)}{P(\vec{d_j}|\overline{R}) \times P(\overline{R})} \sim \frac{P(\vec{d_j}|R)}{P(\vec{d_j}|\overline{R})}$$

- $P(\vec{d_j}|R)$: probability of randomly selecting the document d_j from the set R of relevant documents
- $P(R)$: probability that a document randomly selected from the entire collection is relevant
- $P(\vec{d_j}|\overline{R})$ and $P(\overline{R})$: analogous and complementary

The Ranking

- Assuming independence of index terms

$$sim(d_j, q) \sim \frac{(\prod_{g_i(\vec{d_j})=1} P(k_i|R)) \times (\prod_{g_i(\vec{d_j})=0} P(\overline{k}_i|R))}{(\prod_{g_i(\vec{d_j})=1} P(k_i|\overline{R})) \times (\prod_{g_i(\vec{d_j})=0} P(\overline{k}_i|\overline{R}))}$$

- $P(k_i|R)$: probability that the index term k_i is present in a document randomly selected from the set R of relevant documents
- $P(\overline{k}_i|R)$: probability that the index term k_i is not present in a document randomly selected from the set R
- The probabilities associated with the $\overline{R}$ have meanings which are analogous to the ones just described

The Ranking

- Finally

$$sim(d_j, q) \sim$$

$$\sim \sum_{i=1}^{t} w_{i,q} \times w_{i,j} \times \left(\log \frac{P(k_i|R)}{1 - P(k_i|R)} + \log \frac{1 - P(k_i|\overline{R})}{P(k_i|\overline{R})} \right)$$

- Where
  - $P(\overline{k}_i|R) = 1 - P(k_i|R)$
  - $P(\overline{k}_i|\overline{R}) = 1 - P(k_i|\overline{R})$

The Initial Ranking

- Similarity $sim(d_j, q)$

$$\sum_{i=1}^{t} w_{i,q} \times w_{i,j} \times \left(\log \frac{P(k_i|R)}{1 - P(k_i|R)} + \log \frac{1 - P(k_i|\overline{R})}{P(k_i|\overline{R})} \right)$$

- Probabilities $P(k_i|R)$ and $P(k_i|\overline{R})$?
- Estimates based on assumptions:
  - $P(k_i|R) = 0.5$
  - $P(k_i|\overline{R}) = \frac{n_i}{N}$ where n_i is the number of docs that contain k_i
  - Use this initial guess to retrieve an initial ranking
  - Improve upon this initial ranking

Improving the Initial Ranking

$$\sum_{i=1}^{t} w_{i,q} \times w_{i,j} \times \left(\log \frac{P(k_i|R)}{1 - P(k_i|R)} + \log \frac{1 - P(k_i|\overline{R})}{P(k_i|\overline{R})} \right)$$

- Let
  - V : set of docs initially retrieved
  - V_i : subset of docs retrieved that contain k_i
- Reevaluate estimates:
  - $P(k_i|R) = \frac{V_i}{V}$
  - $P(k_i|\overline{R}) = \frac{n_i - V_i}{N - V}$
- Repeat recursively

Improving the Initial Ranking

$$\sum_{i=1}^{t} w_{i,q} \times w_{i,j} \times \left(\log \frac{P(k_i|R)}{1 - P(k_i|R)} + \log \frac{1 - P(k_i|\overline{R})}{P(k_i|\overline{R})} \right)$$

- To avoid problems with $V = 1$ and $V_i = 0$:
  - $P(k_i|R) = \frac{V_i + 0.5}{V+1}$
  - $P(k_i|\overline{R}) = \frac{n_i - V_i + 0.5}{N - V + 1}$
- Also,
  - $P(k_i|R) = \frac{V_i + \frac{n_i}{N}}{V+1}$
  - $P(k_i|\overline{R}) = \frac{n_i - V_i + \frac{n_i}{N}}{N - V + 1}$

Pluses and Minuses

- Advantages:
  - Docs ranked in decreasing order of probability of relevance
- Disadvantages:
  - need to guess initial estimates for $P(k_i|R)$
  - method does not take into account *tf* and *idf* factors

Brief Comparison of Classic Models

- Boolean model does not provide for partial matches and is considered to be the weakest classic model
- Salton and Buckley did a series of experiments that indicate that, in general, the vector model outperforms the probabilistic model with general collections
- This seems also to be the view of the research community