Modern Information Retrieval

Chapter 2

Modeling

Alternative Probabilistic Models Non-overlapping Lists Proximal Nodes

Alternative Probabilistic Models

Alternative Probabilistic Models

- Three alternative probabilistic models were selected to discuss
 - **BM25**, which is a direct extension of the classic probabilistic model
 - Belief Network Models, which are a direct application of bayesian networks to IR
 - Language Models, which presents a variant of the idea of using the distribution of index terms in the collection as the basis for ranking

Alternative Probabilistic Models BM25 (Best Match 25)

BM25 (Best Match 25)

- BM25 was originated as part of the participation of the Okapi system in the TREC conferences
- A good term weighting is based on three principles
 - inverse document frequency
 - term frequency
 - document length normalization
- The classic probabilistic model covers only the first of these principles
- This reasoning led to a series of experiments on variations of the classic probabilistic model
- Such experimentation culminated on the BM25 ranking formula

At first, the Okapi system used the Equation below as ranking formula

$$sim(d_j, q) \sim \sum_{k_i \in q \land k_i \in d_j} \log \frac{N - n_i + 0.5}{n_i + 0.5}$$

which is the equation used in probabilistic model when no relevance information is provided

It was referred to as the BM1 formula (Best Match 1)

- The next idea was to introduce a term-frequency in BM1 ranking formula
- The used factor was based on a 2-poisson model of term occurrence within the documents:

$$S_1 \times \frac{f_{i,j}}{K_1 + f_{i,j}}$$

- where
 - lacksquare $f_{i,j}$ is the frequency of term k_i within document d_j
 - lacksquare K_1 is a constant setup experimentally for each collection
 - lacksquare S_1 is a scaling constant, normally set to $S_1=(K_1+1)$
- If $K_1 = 0$ (and $S_1 = (K_1 + 1)$), this whole factor becomes equal to 1 and bears no effect in the ranking

- The next step was to introduce document length normalization into the formulation
- This can be attained by changing previous equation to

$$S_1 \times \frac{f_{i,j}}{\frac{K_1 \times len(d_j)}{avg_doclen} + f_{i,j}}$$

where

- $len(d_j)$ is the length of document d_j (computed, for instance, as the number of terms in the document)
- avg_doclen is the average document length for the collection

It was also used a correction factor dependent on document length and on the query length:

$$K_2 \times len(q) \times \frac{avg_doclen - len(d_j)}{avg_doclen + len(d_j)}$$

- where
 - \blacksquare len(q) is the query length (number of terms in the query)
 - $lacksquare K_2$ is a constant
- This factor depends only on the document and query lengths, and was referred as G_2

An additional factor was applied to term frequencies within queries

$$S_3 \times \frac{f_{i,q}}{K_3 + f_{i,q}}$$

- where
 - lacksquare $f_{i,q}$ is the frequency of term k_i within query q
 - $lacksquare K_3$ is a constant
 - S_3 is an scaling constant related to K_3 , normally set to $S_3 = (K_3 + 1)$

- Introduction of these three factors into Equation of BM1 leads to two BM formulas
 - BM15: $sim(d_j,q) \sim$

$$G_2 + \sum_{k_i[q,d_i]} \frac{S_1 f_{i,j}}{(K_1 + f_{i,j})} \times \frac{S_3 f_{i,q}}{(K_3 + f_{i,q})} \times \log \frac{N - n_i + 0.5}{n_i + 0.5}$$

■ BM11: $sim(d_j,q) \sim$

$$G_2 + \sum_{k_i[q,d_i]} \frac{S_1 f_{i,j}}{\frac{K_1 len(d_j)}{avq doclen} + f_{i,j}} \times \frac{S_3 f_{i,q}}{(K_3 + f_{i,q})} \times \log \frac{N - n_i + 0.5}{n_i + 0.5}$$

where $k_i[q,d_j]$ is a short notation for $k_i \in q \land k_i \in d_j$

- Experiments using TREC data indicates that BM11 outperforms BM15
- Some considerations can simplify the previous equations:
 - Empirical evidence suggests a best value of K_2 is 0, which eliminates the G_2 factor from these equations
 - Further, good estimates for the scaling constants S_1 and S_3 are $K_1 + 1$ and $K_3 + 1$, respectively
 - Empirical evidence suggests that making K_3 very large is better
 - For short queries, we can assume that $f_{i,q}$ is 1 for all terms

- These considerations lead to simpler equations as follows
 - BM15: $sim(d_j,q) \sim$

$$\sum_{k_i[q,d_i]} \frac{(K_1+1)f_{i,j}}{(K_1+f_{i,j})} \times \log \frac{N-n_i+0.5}{n_i+0.5}$$

■ BM11: $sim(d_j,q) \sim$

$$\sum_{k_{i}[q,d_{j}]} \frac{(K_{1}+1)f_{i,j}}{\frac{K_{1} len(d_{j})}{avg_doclen} + f_{i,j}} \times \log \frac{N - n_{i} + 0.5}{n_{i} + 0.5}$$

BM25 Ranking Formula

- BM25 was created as a combination of the BM11 and BM15 ranking formulas
- The motivation was to combine the term frequency factors as follows

$$S_1 \times \frac{f_{i,j}}{K_1 \left((1-b) + b \frac{len(d_j)}{avg_doclen} \right) + f_{i,j}}$$

- where b is a constant with values in the interval [0,1]
- If b = 0, the equation reduces to the BM15 term frequency factor
- If b = 1, it reduces to the BM11 term frequency factor
- For values of b between 0 and 1, the equation provides a combination of BM11 with BM15

BM25 Ranking Formula

- In the place of equations BM11 and BM15, we can write
 - BM25: $sim(d_i, q) \sim$

$$\sum_{k_i[q,d_j]} \frac{(K_1+1)f_{i,j}}{K_1((1-b)+b\frac{len(d_j)}{avg_doclen})+f_{i,j}} \times \log \frac{N-n_i+0.5}{n_i+0.5}$$

- where K_1 and b are empirical constants
- $K_1 = 1$ works well with real collections
- b should be kept closer to 1 to emphasize the document length normalization effect present in the BM11 formula
- For instance, b = 0.75 is a reasonable assumption
- Constants values can be fine tunned for particular collections through proper experimentation

BM25 Ranking Formula

- Equation BM15 is nowadays the ranking formula for the BM25 model
- Unlike probabilistic model, the BM25 formula can be computed without relevance information
- There is a consensus that BM25 outperforms classic vector model for general collections
- Thus, it has been used as a baseline for comparison, substituting the vector model

Alternative Probabilistic Models Belief Network Models

- Probability Theory
 - Semantically clear
 - Computationally clumsy
- Why Bayesian Networks?
 - Clear formalism to combine evidences
 - Modularize the world (dependencies)
 - Bayesian Network Models for IR
 - Inference Network (Turtle & Croft, 1991)
 - Belief Network (Ribeiro-Neto & Muntz, 1996)

- Schools of thought in probability
 - frequentist
 - epistemological

- Basic Axioms:
 - $0 \le P(A) \le 1$;
 - P(sure) = 1;
 - $P(A \lor B) = P(A) + P(B)$ if A and B are mutually exclusive

Other formulations

- $P(A) = P(A \land B) + P(A \land \neg B)$
- $P(A) = \sum_{\forall_i} P(A \wedge B_i)$, where B_{i,\forall_i} is a set of exhaustive and mutually exclusive events
- $P(A) + P(\neg A) = 1$
- ightharpoonup P(A|K) belief in A given the knowledge K
- \blacksquare if P(A|B) = P(A), we say: A and B are independent
- if $P(A|B \land C) = P(A|C)$, we say: A and B are conditionally independent, given C
- $P(A \land B) = P(A|B)P(B)$
- $P(A) = \sum_{\forall_i} P(A|B_i) P(B_i)$

Bayes' Rule: the heart of Bayesian techniques

$$P(H|e) = \frac{P(e|H)P(H)}{P(e)}$$

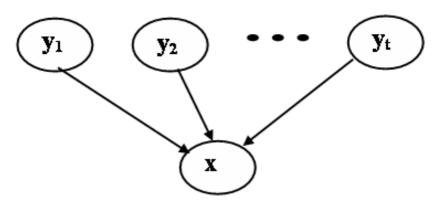
where

- \blacksquare H: a hypothesis and e is an evidence
- ightharpoonup P(H): prior probability
- ightharpoonup P(H|e): posterior probability
- ightharpoonup P(e|H): probability of e if H is true
- \blacksquare P(e): a normalizing constant, then we write:

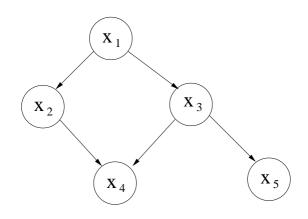
$$P(H|e) \sim P(e|H)P(H)$$

Definition:

Bayesian networks are directed acyclic graphs (DAGS) in which the nodes represent random variables, the arcs portray causal relationships between these variables, and the strengths of these causal influences are expressed by conditional probabilities



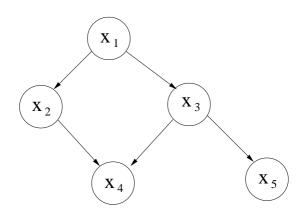
- y_i : parent nodes (in this case, root nodes)
- \mathbf{x} : child node
- y_i cause x
- \blacksquare Y the set of parents of x
- The influence of Y on x can be quantified by any function F(x,Y) such that $\sum_{\forall_x} F(x,Y) = 1$, 0 < F(x,Y) < 1
 - For example, F(x,Y) = P(x|Y)



Given the dependencies declared in a Bayesian Network, the expression for the joint probability can be computed as a product of **local** conditional probabilities, for example,

$$P(x_1, x_2, x_3, x_4, x_5) = P(x_1)P(x_2|x_1)P(x_3|x_1)P(x_4|x_2, x_3)P(x_5|x_3)$$

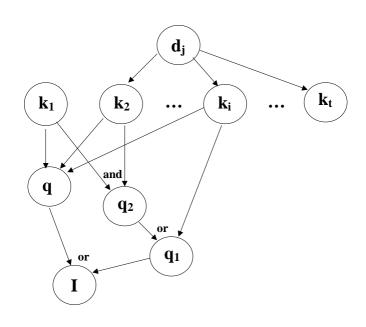
 \blacksquare $P(x_1)$: prior probability of the root node



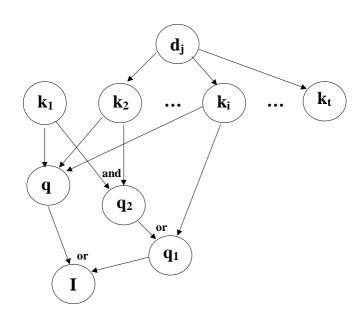
- In a Bayesian network each variable x is conditionally independent of all its non-descendants, given its parents
- For example:

$$P(x_4, x_5 | x_2, x_3) = P(x_4 | x_2, x_3) P(x_5 | x_3)$$

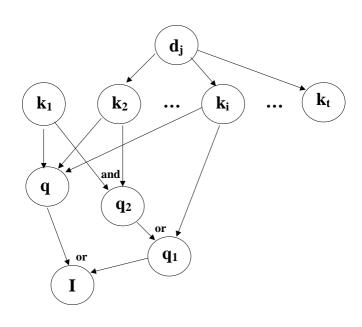
- Epistemological view of the IR problem
- Random variables associated with documents, index terms and queries
- A random variable associated with a document d_j represents the event of observing that document



- Nodes
 - lacksquare documents (d_j)
 - \blacksquare index terms (k_i)
 - \blacksquare queries $(q, q_1, and q_2)$
 - \blacksquare user information need (I)



- Edges
 - If from d_j to its index term nodes k_i indicate that the observation of d_j increase the belief in the variables k_i



- lacksquare d_j has index terms k_2 , k_i , and k_t
 - \blacksquare q has index terms k_1 , k_2 , and k_i
 - \blacksquare q_1 and q_2 model boolean formulation
 - $q_1 = (k_1 \wedge k_2) \vee k_i)$
 - $\blacksquare I = (q \vee q_1)$

Definitions:

- \blacksquare k_1 , d_j , and q random variables
- $\vec{k} = (k_1, k_2, \dots, k_t)$ a t-dimensional vector
- $d_j \in \{0,1\}; q \in \{0,1\}$
- the ranking of a document d_j is computed as $P(q \wedge d_j)$ where q and d_j are short representations for q=1 and $d_j=1$ (d_j stands for a state where $d_j=1$ and $\forall_{l\neq j}\Rightarrow d_l=0$, because we observe one document at a time)

$$P(q \wedge d_j) = \sum_{\forall \vec{k}} P(q \wedge d_j | \vec{k}) \times P(\vec{k})$$

$$= \sum_{\forall \vec{k}} P(q \wedge d_j \wedge \vec{k})$$

$$= \sum_{\forall \vec{k}} P(q | d_j \times \vec{k}) \times P(d_j \times \vec{k})$$

$$= \sum_{\forall \vec{k}} P(q | \vec{k}) \times P(\vec{k} | d_j) \times P(d_j)$$

$$P(\overline{q \wedge d_j}) = 1 - P(q \wedge d_j)$$

As the instantiation of d_j makes all index term nodes mutually independent $P(k|d_j)$ can be a product, then

$$P(q \wedge d_{j}) = \sum_{\forall \vec{k}} P(q|\vec{k}) \times \left(\prod_{\forall i|g_{i}(\vec{k})=1} P(k_{i}|d_{j}) \times \prod_{\forall i|g_{i}(\vec{k})=0} P(\overline{k}_{i}|d_{j}) \right) \times P(d_{j})$$

$$P(\overline{q \wedge d_{j}}) = 1 - P(q \wedge d_{j})$$

- The prior probability $P(d_j)$ reflects the probability associated to the event of observing a given document d_j
 - \blacksquare Uniformly for N documents
 - $P(d_j) = \frac{1}{N}$
 - $P(\overline{d}_j) = 1 \frac{1}{N}$
 - **Based** on norm of the vector d_j
 - $P(d_j) = \frac{1}{|\vec{d_j}|}$
 - $P(\overline{d}_j) = 1 P(d_j)$

For the Boolean Model

$$P(d_j) = \frac{1}{N}$$

$$P(\overline{d}_j) = 1 - P(d_j)$$

$$P(k_i|d_j) = \begin{cases} 1 & \text{if } g_i(d_j) = 1 \\ 0 & \text{otherwise} \end{cases}$$

 $P(\overline{k}_i|d_j) = 1 - P(k_i|d_j)$

 \Rightarrow only nodes associated with the index terms of the document d_j are activated

For the Boolean Model

$$P(q|\vec{k}) = \begin{cases} 1 & \text{if } \exists \vec{q}_{cc} \mid (\vec{q}_{cc} \in \vec{q}_{dnf}) \land (\forall k_i, g_i(\vec{k}) = g_i(\vec{q}_{cc})) \\ 0 & \text{otherwise} \end{cases}$$

$$P(\overline{q}|\vec{k}) = 1 - P(q|\vec{k})$$

⇒ one of the conjunctive components of the query must be matched by the active index terms in k

For a *tf-idf* ranking strategy

$$P(d_j) = \frac{1}{|\vec{d_j}|}$$

$$P(\overline{d_j}) = 1 - P(d_j)$$

⇒ prior probability reflects the importance of document normalization

For a *tf-idf* ranking strategy

$$P(k_i|d_j) = f_{i,j}$$

$$P(\overline{k}_i|d_j) = 1 - P(k_i|d_j)$$

 \Rightarrow the relevance of the a index term k_i is determined by its normalized term-frequency factor $f_{i,j} = \frac{freq_{i,j}}{maxfreq_{i,j}}$

For a *tf-idf* ranking strategy

Define a vector k_i given by

$$\vec{k}_i = \vec{k} \mid (g_i(\vec{k}) = 1 \land \forall_{j \neq i} \ g_j(\vec{k}) = 0)$$

 \Rightarrow in the state k_i only the node k_i is active and all the others are inactive

For a tf-idf ranking strategy

$$P(q|\vec{k}) = \begin{cases} idf_i & \text{if } \vec{k} = \vec{k}_i \land g_i(\vec{q}) = 1\\ 0 & \text{if } \vec{k} \neq \vec{k}_i \lor g_i(\vec{q}) = 0 \end{cases}$$

$$P(\overline{q}|\vec{k}) = 1 - P(q|\vec{k})$$

⇒ we can sum up the individual contributions of each index term by its normalized idf

For a tf-idf ranking strategy

As $P(q|\vec{k}) = 0$ if $\vec{k} \neq \vec{k}_i$, we can rewrite $P(q \wedge d_j)$ as

$$P(q \wedge d_j) = \sum_{\forall \vec{k}_i} P(q|\vec{k}_i) \times P(k_i|d_j) \times \left(\prod_{\forall l \neq i} P(\overline{k}_l|d_j)\right) \times P(d_j)$$

$$= \left(\prod_{\forall i} P(\overline{k}_i|d_j)\right) \times P(d_j) \times \sum_{\forall \vec{k}_i} P(k_i|d_j) \times P(q|\vec{k}_i) \times \frac{1}{P(\overline{k}_i|d_j)}$$

For a *tf-idf* ranking strategy

Applying the previous probabilities we have

$$P(q \wedge d_j) = C_j \times \frac{1}{|\vec{d_j}|} \times \sum_{\forall i | g_i(\vec{d_j}) = 1 \wedge g_i(\vec{q}) = 1} f_{i,j} \times i df_i \times \frac{1}{1 - f_{i,j}}$$

- $\Rightarrow C_j$ vary from document to document
- ⇒ the ranking is distinct of the one provided by the vector model

Combining evidential source

Let
$$I = q \vee q_1$$

$$P(I \wedge d_j) = \sum_{\vec{k}} P(I|\vec{k}) \times P(\vec{k}|d_j) \times P(d_j)$$

$$= \sum_{\vec{k}} (1 - P(\overline{q}|\vec{k}) P(\overline{q}_1|\vec{k})) \times P(\vec{k}|d_j) \times P(d_j)$$

⇒ it might yield a retrieval performance which surpasses the retrieval performance of the query nodes in isolation (Turtle & Croft)

- As the Inference Network Model
 - Epistemological view of the IR problem
 - Random variables associated with documents, index terms and queries
- Contrary to the Inference Network Model
 - Clearly defined sample space
 - Set-theoretic view
 - Different network topology

- The Probability Space
 - Define:
 - $K = \{k_1, \dots, k_t\}$ the sample space (a concept space)
 - $u \subset K$ a subset of K (a concept)
 - \bullet k_i an index term (an elementary concept)
 - $\vec{k} = \{\vec{k}_1, \vec{k}_2, ..., \vec{k}_t\}$ a vector associated to each u such that $g_i(\vec{k}) = 1 \iff k_i \in u$
 - k_i a binary random variable associated with the index term k_i , $(k_i = 1 \iff g_i(\vec{k}) = 1 \iff k_i \in u)$

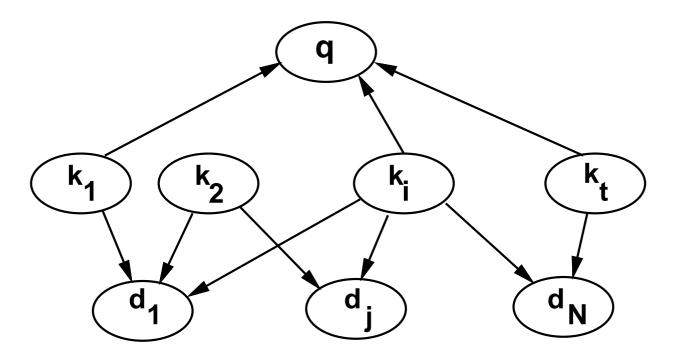
- A Set-Theoretic View
 - Define:
 - \blacksquare a document d_j and query q as concepts in K
 - \blacksquare a generic concept c in K
 - \blacksquare a probability distribution P over K, as

$$P(c) = \sum_{u} P(c|u) \times P(u)$$

$$P(u) = \left(\frac{1}{2}\right)^{t}$$

 \blacksquare P(c) is the *degree of coverage* of the space K by c

Network topology



- Assumption
 - $P(d_j|q)$ is adopted as the rank of the document d_j with respect to the query q. It reflects the degree of coverage provided to the concept d_j by the concept q

\blacksquare The rank of d_j

$$P(d_{j}|q) = P(d_{j} \wedge q)/P(q)$$

$$P(d_{j}|q) \sim P(d_{j} \wedge q)$$

$$P(d_{j}|q) \sim \sum_{\forall u} P(d_{j} \wedge q|u) \times P(u)$$

$$P(d_{j}|q) \sim \sum_{\forall u} P(d_{j}|u) \times P(q|u) \times P(u)$$

$$P(d_{j}|q) \sim \sum_{\forall \vec{k}} P(d_{j}|\vec{k}) \times P(q|\vec{k}) \times P(\vec{k})$$

- For the vector model
 - \blacksquare Define a vector k_i given by

$$\vec{k}_i = \vec{k} \mid (g_i(\vec{k}) = 1 \land \forall_{j \neq i} \ g_j(\vec{k}) = 0)$$

 \Rightarrow in the state k_i only the node k_i is active and all the others are inactive

- For the vector model
 - Define

$$P(q|\vec{k}) = \begin{cases} \frac{w_{i,q}}{\sqrt{\sum_{i=1}^{t} w_{i,q}^2}} & \text{if } \vec{k} = \vec{k}_i \land g_i(q) = 1\\ 0 & \text{otherwise} \end{cases}$$

$$P(\overline{q}|\vec{k}) = 1 - P(q|\vec{k})$$

 $\Rightarrow \frac{w_{i,q}}{\sqrt{\sum_{i=1}^t w_{i,q}^2}}$ is a normalized version of weight of the index term k_i in the query q

- For the vector model
 - Define

$$P(d_j|\vec{k}) = \begin{cases} \frac{w_{i,j}}{\sqrt{\sum_{i=1}^t w_{i,j}^2}} & \text{if } \vec{k} = \vec{k}_i \ \land \ g_i(\vec{d}_j) = 1 \\ 0 & \text{otherwise} \end{cases}$$

$$P(\overline{d_j}|\vec{k}) = 1 - P(d_j|\vec{k})$$

 $\Rightarrow \frac{w_{i,j}}{\sqrt{\sum_{i=1}^t w_{i,j}^2}}$ is a normalized version of the weight of the index term k_i in the document d_i

Bayesian Network Models

Comparison

- Inference Network Model is the first and well known
- Belief Network adopts a set-theoretic view
- Belief Network adopts a clearly define sample space
- Belief Network provides a separation between query and document portions
- Belief Network is able to reproduce any ranking produced by the Inference Network while the converse is not true (for example: the ranking of the standard vector model)

Bayesian Network Models

- Computational costs
 - Inference Network Model one document node at a time then is linear on number of documents
 - Belief Network only the states that activate each query term are considered
 - The networks do not impose additional costs because the networks do not include cycles

Bayesian Network Models

- Impact
 - The major strength is net combination of distinct evidential sources to support the rank of a given document

Structured Text Models

Introduction

- Keyword-based query answering considers that the documents are flat, i.e., a word in the title has the same weight as a word in the body of the document
- But, the document structure is one additional piece of information which can be taken advantage of
- For instance, words appearing in the title or in sub-titles within the document could receive higher

Introduction

- Consider the following information need:
 - Retrieve all documents which contain a page in which the string "atomic holocaust" appears in italic in the text surrounding a Figure whose label contains the word earth
- The corresponding query could be:
 - same-page(near("atomic holocaust", Figure(label("earth"))))

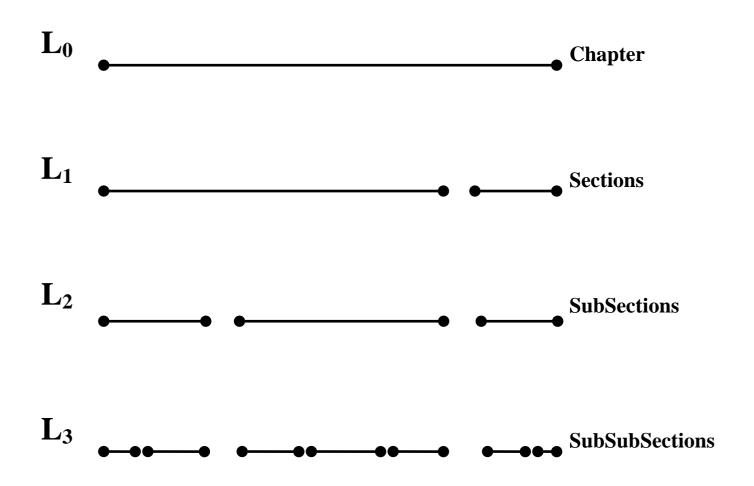
Introduction

- Advanced interfaces that facilitate the specification of the structure are also highly desirable
- Models which allow combining information on text content with information on document structure are called structured text models
- Structured text models include no ranking (open research problem)

Basic Definitions

- Match point: the position in the text of a sequence of words that match the query
 - Query: "atomic holocaust in Hiroshima"
 - **Doc** d_j : contains 3 lines with this string
 - Then, doc d_j contains 3 match points
- Region: a contiguous portion of the text
- Node: a structural component of the text such as a chapter, a section, etc.

- Due to Burkowski, 1992
- Idea: divide the text in non-overlapping regions which are collected in a list
- Multiple ways to divide the text in non-overlapping parts yield multiple lists:
 - a list for chapters
 - a list for sections
 - a list for subsections
- Text regions from distinct lists might overlap



- Implementation:
 - single inverted file that combines keywords and text regions
 - to each entry in this inverted file is associated a list of text regions
 - lists of text regions can be merged with lists of keywords

- Regions are non-overlapping which limits the queries that can be asked
- Types of queries:
 - select a region that contains a given word
 - select a region A that does not contain a region B (regions A and B belong to distinct lists)
 - select a region not contained within any other region

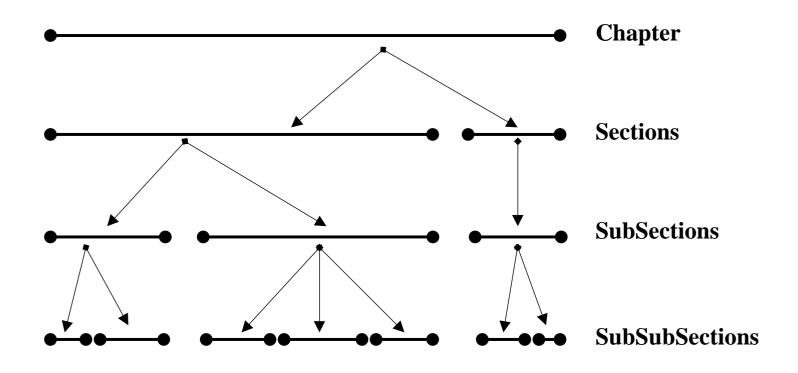
Conclusions

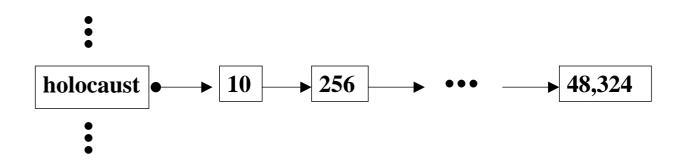
- The non-overlapping lists model is simple and allows efficient implementation
- But, types of queries that can be asked are limited
- Also, model does not include any provision for ranking the documents by degree of similarity to the query
- What does structural similarity mean?

- Due to Navarro and Baeza-Yates, 1997
- Idea: define a strict hierarchical index over the text.
 This enrichs the previous model that used flat lists
- Multiple index hierarchies might be defined
- Two distinct index hierarchies might refer to text regions that overlap

Definitions

- Each indexing structure is a strict hierarchy composed of
 - chapters
 - sections
 - subsections
 - paragraphs
 - lines
- Each of these components is called a *node*
- To each node is associated a text region





- Key points:
 - In the hierarchical index, one node might be contained within another node
 - But, two nodes of a same hierarchy cannot overlap
 - The inverted list for keywords complements the hierarchical index
 - The implementation here is more complex than that for non-overlapping lists

- Queries are now regular expressions:
 - search for strings
 - references to structural components
 - combination of these
- Model is a compromise between expressiveness and efficiency
- Queries are simple but can be processed efficiently
- Further, model is more expressive than non-overlapping lists

- Query: find the sections, the subsections, and the subsubsections that contain the word "holocaust"
 - .[(*section) with ("holocaust")]
- Simple query processing:
 - traverse the inverted list for "holocaust" and determine all match points
 - use the match points to search in the hierarchical index for the structural components

- Query: [(*section) with ("holocaust")]
- Sophisticated query processing:
 - get the first entry in the inverted list for "holocaust"
 - use this match point to search in the hierarchical index for the structural components
 - Innermost matching component: smaller one
 - Check if innermost matching component includes the second entry in the inverted list for "holocaust"
 - If it does, check the third entry and so on
 - This allows matching efficiently the nearby (or proximal) nodes

Conclusions

- Model allows formulating queries that are more sophisticated than those allowed by non-overlapping lists
- To speed up query processing, nearby nodes are inspected
- Types of queries that can be asked are somewhat limited (all nodes in the answer must come from a same index hierarchy!)
- Model is a compromise between efficiency and expressiveness