Generalized Optical Flow Model for Scattering Media

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Abstract—This paper proposes a novel methodology to estimate the optical flow in scattering media, which consists on new formulation based on the classical Horn-Schunk approach and the optical image formation model. Our formulation is able to deal with the hard problem of tracking points in a medium where there is absorption and scattering effects. This approach generalizes assumptions of the Horn-Schunk model in order to tackle both non-scattering and scattering media. Our approach uses the Dark Channel Prior to estimate the scene transmission, which attains a significant improvement in the optical flow estimation in scattering media. We show that our approach outperformed state-of-the-art models and we provide a detailed analysis of our technique that shows its applicability to image sequences acquired both in simulated and real scenes.

I. INTRODUCTION

The apparent motion of rigid objects in a scene may be inferred by the motion field obtained from an image sequence acquired by a camera. One of the usual assumptions is that the relative motion would be described by a single rigid motion and that scene illumination conditions do not vary significantly. A more restrictive assumption requires that all objects move with the same motion. This last restriction may be weakened if the image may be segmented into areas capturing similarly moving objects. The motion field is defined as the velocity of moving image points estimated from a sequence of images acquired by a static camera from a moving scene or form a moving camera from a static scene. This projection of the 3-D velocity field onto the 2-D image plane is usually computed from the variation of image brightness between consecutive frames, and it called the optical flow \cite{1}.

Optical flow estimation is an important stage to recover the spatial motion from a sequence of images, and it has been increasingly used in the past years with great success, despite its dependence on scene texture and illumination constancy. In scattering media, e.g. underwater or under foggy conditions, light propagation may be affected by absorption and scattering effects before it reaches the camera, causing what is generally termed haze. Hazy images are most challenging to optical flow estimation due to the overall reduction in contrast, increased blur and color shift, which directly affects visibility. This phenomenon is strongly dependent on the distance between the image plane and the scene. In other words, the pernicious effect of haze on the image increases with the distance between objects and the camera. Thus, the typical constant illumination assumption for computing the optical flow quickly breaks down in scattering media even for small relative displacements.

Knowing the scene transmission is crucial to the computation of the optical flow using the proposed model. However, the transmission estimation is not trivial, specially if only a single image is available. There are numerous methods to estimate the scene transmission based on multiple images: Stereo \cite{2,3}; polarization filters \cite{4} or other assumptions and constraints \cite{5,6,7}. However, this estimation using a single image is a severely ill-posed problem. Recent developments by He et al. \cite{8,9}, called Dark Channel Prior (DCP), enables the estimation of scene transmission in scattering media from a single image. Their approach is based on the observations that haze-free images have at least one color in the RGB spectrum with low intensity value. More recently, an adaptation of the DCP to underwater environments was proposed \cite{10}.

The main contribution of the paper is a new optical flow formulation that broadens the use of the Horn-Schunk technique \cite{1} with the inclusion of an optical model of scattering media. We call it Generalized Optical Flow Model (GOFM). It is based on a simplification of the Jaffe-McGlamerly model \cite{11,12}, and it is similar to the classic model due to Koschmieder \cite{13}. This new formulation extends the previous one, allowing the optical flow estimation in scattering and non-scattering media. For non-scattering media, the model is simplified to become the Horn-Schunk model, one the most widely used in the literature \cite{14}. The proposed model uses the scene transmission in order to better estimate the optical flow in scattering media.

II. OPTICAL MODEL

Images captured from scene immersed in scattering media can be modeled as a complex interaction between the light, the medium and the scene structure. One of the most popular models used in the literature was proposed by Jaffe-McGlamerly \cite{11,12}. Their model is composed of three main components: Direct illumination ($I_d$), forward-scattering ($I_{fs}$) and backscattering ($I_{bs}$), as shown in Eq. 1:

$$I = I_d + I_{fs} + I_{bs}. \quad (1)$$

The direct component is the fraction of light that reaches the camera. Part of the light that radiates from object is lost due to scattering and absorption. The direct component captures this effect. According to Schechner and Karpe1 \cite{4}, backscattering is the prime reason for image contrast degradation, and for this reason the forward scattering can be usually neglected. Indeed, this assumption is valid when the average depth between the scene and the camera is large. After simplifications and
Following the nomenclature used in [8], the direct component is given by:

\[ I_d = J e^{-\eta d} = J t_r, \]  

where \( J \) and \( d \) are the radiances and the depth, respectively; \( \eta \) is the attenuation coefficient (frequently \( c \) in the literature) and \( t_r \) is the scene transmission, considered as the exponential term.

In foggy scenes, the attenuation coefficient \( \eta \) is composed only by the scattering coefficient, called \( \beta \) [8]. In underwater environments, the absorption represents the inherent properties of the medium [12], thus the coefficient \( \eta \) is the sum of the absorption, \( a \), and the scattering, \( \beta \), coefficients, both being wavelength dependent. Depth can be estimated using the transmission \( t \), but the constant \( c \) is usually unknown. However, it can be measured by using a turbidimeter or estimated by a calibration procedure [3].

The backscattering component is not due to the object’s radiance, but it results from the interaction between the ambient illumination sources and particles dispersed in the medium. Therefore, a simplified model is defined by Eq. 3:

\[ I_{bs} = A(1 - e^{-\eta d}) = A(1 - t_r), \]  

where \( A \) is the airlight or the global light in the scene, also called \( B_{ic} \) in the underwater literature. This term is a scalar that depends on the wavelength. The airlight constant \( A \) can be estimated in different ways: Using calibration [15], finding the farthest pixel in the scene [3], [2], finding the brightest pixel in the image [16], [5] or finding the brightest pixel in the dark channel [8], [9]. It can also be modeled as a function \( A(x, y) \) and estimated using a simple calibration process [17] in order to enable the model to deal with non-uniform illumination, a common situation in underwater environment.

Therefore, an enhanced model that describes the image formation in participating medium can be stated as:

\[ I(x) = J(x) t_r(x) + A(1 - t_r(x)), \]  

where \( x \) are the pixel coordinates \((x, y)\) and \( I(x) \) is the image obtained in the scattering media. This model is valid assuming that the medium is homogeneous. In this simplified version, it is very similar to the model proposed by Koschmieder [13]. This model is valid for both media, scattering and non-scattering. In the case of non-scattering media, the attenuation coefficient is approximately zero, thus the scene transmission is equal to one. In this case, the model is invariant to the distance between the camera and the scene, as originally expected.

Once \( t_r(x) \) and \( A \) are estimated, the radiances can be recovered using Eq. 5 based on Eq. 4.

\[ J(x) = \frac{I(x) - A}{t_r(x)} + A. \]  

III. STANDARD OPTICAL FLOW

There is a large body of research in the field of optical flow and in the last 30 years we have witness impressive advances in this area [14]. The majority of these methods strongly resembles the original formulation proposed by Horn and Schunk [1], which assumes constancy in the brightness patterns in the image. Let \( I(x, y, t) \) be the image brightness at pixel \((x, y)\) on image plane at time \( t \). Considering that the brightness of every point remains constant, the variation of pixel intensity in time is given by:

\[ \frac{dI}{dt} = \frac{\partial I}{\partial x} \frac{dx}{dt} + \frac{\partial I}{\partial y} \frac{dy}{dt} + \frac{\partial I}{\partial t} = 0. \]  

(6)

The partial derivatives can be easily estimated from the images. Assuming \( \frac{dx}{dt} = u \) and \( \frac{dy}{dt} = v \), we can rewrite Equation 6 as:

\[ I_x u + I_y v + I_t = 0. \]  

(7)

Since we have one Equation 7 per pixel, it is not possible to solve this system without including an additional constraint. Different constraints can be assumed such as: Global smoothness [1], local smoothness [18], texture constancy [19], and multiple color channel [20].

Although works such as [21], [22] and [23] have proposed to deal with the optical flow estimation in outdoor environments, they did not address the optical model and the relation between brightness and contrast with the depths of the scene. In general, the optical effects are handled as a variation in the illumination, contrast or brightness of a pixel, leaving out the scattering and the absorption effects.

A. Optical Flow in Scattering Media

In order to tackle the scattering and absorption affects in underwater images to estimate the optical flow, [19] proposed a method that combines intensities and texture. Despite the good results in real underwater images, these were shown only for images acquired from scenes near to the camera.

Negahdaripour and Madjidi [24], [25] proposed a method to estimate optical flow in underwater environment. In their work scattering and absorption effects were modeled simple version of Jaffe-McGlamery model [11], [12] where only the direct component is considered. In fact, the model is only used to get insights about the color space to be used and the applicability of the model. The authors also worked on the HSV color space and extended a model previously proposed in [21]. Their model based on GDIM (Generalized Dynamic Image Model) is described by Equation 8:

\[ I_x u + I_y v + I_t - Im = 0, \]  

(8)

where \( Im \) is a new radiometric term, which takes into account the brightness variation of a scene point between the two views.

Even though this model relaxes the brightness constancy assumption, it does not consider the influence of the distance between scene and camera, which is responsible for a significant impact on the quality of images acquired in scattering media.

IV. GENERALIZED OPTICAL FLOW MODEL - GOFM

In spite of advances in optical flow in past years, virtually all proposed methodologies are not catered for scattering media, since the main assumption of Horn and Schunk’s model [1] is not valid in these environments. As described by the model in Eq. 4, the brightness in the image \( I(x, y, t) \) is not constant due to effects of the transmission.
Nevertheless, one may assume that the radiance \( J(x, y, t) \) is approximately constant. Considering this assumption, we can differentiate a generalized model in order to obtain the optical flow from images acquired in scattering media. We call this new model Generalized Optical Flow Model (GOFM) and it is given by:

\[
\frac{dJ}{dt} = \frac{\partial J}{\partial x} \frac{dx}{dt} + \frac{\partial J}{\partial y} \frac{dy}{dt} + \frac{\partial J}{\partial t} = 0, \tag{9}
\]

where the radiance \( J(x, y, t) \) is given by Eq. 5.

The partial derivatives \( \frac{\partial J}{\partial x} \), \( \frac{\partial J}{\partial y} \) and \( \frac{\partial J}{\partial t} \) are computed as:

\[
\frac{\partial J}{\partial x} = \frac{1}{r} \frac{\partial J}{\partial x} + (I - A) \frac{\partial J}{\partial x}, \tag{10}
\]

\[
\frac{\partial J}{\partial y} = \frac{1}{r} \frac{\partial J}{\partial y} + (I - A) \frac{\partial J}{\partial y}, \tag{11}
\]

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\frac{\partial J}{\partial t} = \frac{1}{r} \frac{\partial J}{\partial t} + (I - A) \frac{\partial J}{\partial t}. \tag{12}
\]

Finally, applying the eqs. 10, 11, and 12 into Eq. 9, we obtain the generalized optical flow model defined by:

\[
t_i(I_xu + I_yv + I_t) + (I - A)(T_x u + T_y v + T_t) = 0, \tag{13}
\]

where the image derivatives can be estimated as proposed by Sun et al. [14], \( t_i = t_r^{-1} \) is the inverse of the scene transmission and \( T_x \), \( T_y \) and \( T_t \) are the inverse medium transmission derivatives.

In non-scattering media, the attenuation coefficient, \( \eta \), is approximately zero, thus the medium transmission is an unitary matrix and the derivatives \( T \) are zero. In this way, the model obtained is a generalization of the brightness constancy model proposed by Horn and Schunk [1], defined in Eq. 7.

In order to improve the numerical stability of the model, we have used \( t_i = e^{\eta d(x,y,t)} = e^{D(x,y,t)} \). Thus, the new function is estimated as \( D(x, y, t) = -\log t_i(x, y, t) \). By using this assumption, we may write the final version of the model as:

\[
(I_x u + I_y v + I_t^c) + (I - A)(D_x u + D_y v + D_t^c) = 0. \tag{14}
\]

where the derivatives of function \( D(x, y, t) \) in relation to \( x \), \( y \) and \( t \) are defined as \( D_x \), \( D_y \) and \( D_t \), respectively. The index \( c \) represents each channel in the RGB representation.

In scattering media, specially in underwater environments, the spectral characteristics of the medium can modify the three channel contents significantly, to the point that this may provide valuable information for the optical flow estimation [25]. Thus, the model is independently defined for each color channel that is similar to proposed by Ohta [20] for non-scattering media.

In our model, we assumed both transmission and global light as known parameters. In our tests, the transmission is estimated using the DCP method [8]. For underwater sequences, transmission is estimated using UDCP [10]. Moreover, the global light is estimated by finding the brightest pixel in the dark channel [8].

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We performed several experiments to validate and to evaluate the robustness of our methodology using simulated and real image sequences. Furthermore, we performed several comparison against three different methods. Considering the model of the Eq. 7, both global and local smoothness constraints approaches are evaluated using the classical Horn Schunk (HS) [1] and the Lucas Kanade (KN) [18], respectively. In order to compare with a state-of-art optical flow estimation method in scattering media, the method proposed by Madjidi and Negahdaripour (MN) [24], [25] was chosen due its relevance in optical flow estimation in scattering media.

The ground truth estimation was obtained using the method proposed by Sun et al. [14] in the non-scattering media images. This method is one of the best ranked in the Middlebury Optical Flow Dataset [26]. Additionally, a manual refinement was performed in order to improve the ground truth.

Qualitative evaluation follows the same approach proposed in [26], where the absolute endpoint error is estimated along with the average angular error proposed by Barron et al. [27]. The endpoint error is estimated in pixels while the angular error is estimated in degrees.

A. Implementation

The implementations used in experiments are based on the work of Barron et al. [27] and Sun et al. [14] that provides many useful and simple suggestions about the practical aspects of the optical flow estimation problem. Initially, we smoothed all images using a Gaussian kernel with different values of \( \sigma \). We defined \( \sigma = 1.5 \) for the simulated images and \( \sigma = 2.5 \) for the real images. The first-order derivatives of the image functions are estimated using five-points kernel to compute each derivative. Finally, the flow field is post-processed removing the boundary pixels (10 pixels) and using \( 5 \times 5 \) median filter.

In order to compare the different methods, we use the same parameters for all methods. In the case of Horn and Schunk [1], we use the parameters values as defined by the authors, e.g. \( \alpha = 1 \). In the other methods, we use a local window of \( 11 \times 11 \) pixels [25]. All methods are implemented using the simplest model without optimization and multiple scale representation, since the objective here is to compare the optical flow model in similar conditions, instead of the best possible performance.

For our model, we need to obtain the scene transmission. This is estimated using a version of the DCP method [8], [9] that we implemented following the original method. For underwater sequences, transmission is estimated using an adaptation...
UDCP [10]. In all of these methods, the local patch size was $\Omega = 15 \times 15$ pixels and the removal amount $\omega = 0.95$, both values as suggested by the original authors. Finally, the scene transmission estimated by the DCP/UDCP was refined using a soft matting algorithm [28].

B. Simulated Results

The simulated results were obtained using two images from the Middlebury Stereo Dataset - Rocks1 [29], where the disparity maps are available. Similar to previous works in scattering media [4], [10], the model defined in Eq. 4 was applied in both images in order to simulate the underwater effects. We defined the attenuation coefficient $\eta$ as a function of the chlorophyll concentration $C$ as proposed by Mobley [30] to simulate oceanic water. The constant $C$ is called in the present work turbidity level as in [25]. The depth map is obtained using previous knowledge about stereo baseline and focal distance that is multiplied by a 1.5 factor in order to increase the global distance and depth variation. The scene is placed away from the camera ($\approx 4m; \approx 6.5m$). We use $C = [0, 1]$ in the simulation, where $C = 0$ represents the clean water.

Fig. 1 shows the results obtained using the simulated sequence. Fig. 1(a) shows the image from the Middlebury Stereo Dataset - Rocks1 [29] before we applied the underwater effect. A simulated image is shown in Fig. 1(b), where the water turbidity level used was $C = 0.5$. A qualitative evaluation between all four methods can be performed on the figs. 1(c) and 1(d). The average angular error shows the advantage of the proposed methodology in comparison to the HS[1] and MN[25], although all methods increase the error proportionally to the turbidity level. Similar to the angular error, the endpoint error provided by LK[18] and the proposed method (GOFM) are comparable, with a small advantage to the latter. All methods obtain results that are similar in terms of endpoint error. Due to the small depth variation in the image sequence, the improvement of the proposed method becomes limited. But even in this situation, the GOFM shows better results than the others. Furthermore, the large movement between images affects the performance of all methods because a single scale estimation is used.

C. Results with Real Scenes

We also used image sequences from real scenes to evaluate our methodology. Firstly, an outdoor sequence under fog conditions acquired by He et al. [9] was used to compare our method against several standard optical flow approaches (a frame of the sequence used is shown in Fig. 2). Initially, we randomly selected a frame from the sequence, and then we estimate the ground truth between this restored frame and a restored sequence of six consecutive frames by using the previously detailed methodology. Therefore, we computed the optical flow using HS[1], LK[18] and MN[25] for the sequences, original and restored. The results from our method are estimated using the original sequence under fog condition, where the DCP [8] provides the scene transmission for each frame in the sequence. Figs. 2(a) and 2(b) show a sample from the original image sequence and the restored one.

Figs. 3(a) and 3(b) show the average angular error and the endpoint error obtained by all methods using the original image sequence. For a small displacement between images, the results are similar with a small advantage to the LK[18]. However, the GOFM becomes dominant and provides the best results when the displacement increases (e.g. in fast motions), mainly in terms of angular error. It is interesting to see that the results obtained in the restored image sequence by HS[1], LK[18] and MN[25] are worse than the GOFM, even though it is applied in the scattering image (figs. 3(c) and 3(d)). However, the restored sequence error is slightly smaller than the scattering media sequence. In terms of endpoint error, the GOFM presents the best results in all different sequences.

Real results are also obtained using underwater image sequences, as shown in Fig. 4. A sample image from the dataset is shown in Fig. 4(a). The optical flow fields are estimated using LK[18], MN[25] and HS[1], figs. 4(d), 4(b) and 4(c), respectively. The consecutive frames are selected from the image sequence for which the movement is approximately perpendicular to the image plane with a small counterclockwise rotation in this axis. This makes easier to understand the result. The optical flow estimation provided by our method is shown in figs. 4(e) and 4(f), where the model is applied only to the intensity image and in the three RGB color channels, Eq. 14. The UDCP [10] is applied in order to obtain the scene transmission. LK[18] and MN[25] are prone
to error in the top region of the image, due to the presence of haze in the sequence. On the other hand, the HS[1] and our method produce more realistic results. It is possible to see that the HS fails in the bottom-right region, where some vectors point to the right, when the correct would be to the left. The use of color information improves the optical flow density, as it can be perceived in figs. 4(e) and 4(f). Moreover, the fish movement is correctly estimated by our method (red box in figs. 4(e) and 4(f)), whereas the HS[1] fails in the bottom-right region, where some vectors are incorrect. This shows the limited application of the HS[1] in fog situations.

The present paper proposes a novel methodology in order to estimate optical flow in scattering media. It is based on the additional information provided by image acquired in this kind of media that allows us to obtain a depth estimation even for a single image. We evaluated this new constraint in simulated and real outdoor under fog and underwater image sequences, being this last one the most challenging type of scattering media. The results show an important gain in terms of endpoint and average angular errors, even for a restored image. Our method has shown a better performance in comparison with LK and HS, with the LK presenting better results for short-range motions and the HS for large range motion. The state-of-art MN is only competitive in restored images in fog situation that shows its limited application.

The problem of optical flow in scattering media needs to be further investigated. Following the recent success obtained by the Middlebury Optical Flow Dataset [26], one important step would be the generation of a dataset of scattering media scenes where the ground truth is available, both for optical flow and scene transmission. This would enable the testing and comparison of new techniques. We intend to improve the present method using coarse-to-fine estimation using multiple scales, and testing other optimization methods that provides more competitive results compared with other modern approaches. We also intend to relax the priori knowledge about the transmission, we will investigate the simultaneous estimation of optical flow and transmission.

VI. CONCLUSIONS

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REFERENCES

Fig. 4. Real image sequence in underwater scene with images acquired in Guanacaste Coast Pacific - Costa Rica [31]: (a) Sample image with a fish marked by a red box; (b,c,d) Optical flow fields obtained using LK[18], MN[25] and HS[1], respectively; and (e,f) Optical flow fields obtained using GOFM using only intensity image and color image, respectively. In these fields, it is possible to see that the proposed method is able to estimate the fish’s movement, even under strong haze.