

Consider the program below, written in SML/NJ. This program contains five routines that operate on lists. The first, `halve`, splits a list into two balanced halves. Two of the other routines, `balanced` and `sublist`, are predicates, that is, functions that return booleans, based on some condition.

```
01 fun halve nil = (nil, nil)
02   | halve [a] = ([a], nil)
03   | halve (a::b::cs) =
04     let
05       val (x, y) = halve cs
06     in
07       (a::x, b::y)
08     end
09
10 fun len nil = 0
11   | len (_::t) = 1 + len t
12
13 fun balanced L0 L1 = abs(len(L0) - len(L1)) <= 1
14
15 fun contains e nil = false
16   | contains e (x::t) = e = x orelse contains e t
17
18 fun sublist nil _ = true
19   | sublist (h::t) L = contains L h andalso sublist t L
```

**Figure 1:** Three programs written in SML/NJ that operate on lists.

1. Prove that if `halve L = (L0, L1)`, then `balanced L0 L1`
2. Prove that if `halve L = (L0, L1)`, then `sublist L0 L1`
3. Prove that if `halve L = (L0, L1)`, then `sublist L1 L0`

