

Below we define a few relations. The first two relations, the natural numbers, are a representation of two of Peano's Axioms for Natural Numbers:

**[z]:** number(z)

**[s]:** 
$$\frac{\text{number}(N)}{\text{number}(\text{succ } N)}$$

**Figure 1:** The natural numbers

**[even\_z]:** even(z)

**[even\_s]:** 
$$\frac{\text{even}(N)}{\text{even}(\text{succ } (\text{succ } N))}$$

**Figure 2:** The even numbers

**[plus\_z]:** plus(z, N, N)

**[plus\_s]:** 
$$\frac{\text{plus}(N1, N2, N3)}{\text{plus}(\text{succ } N1, N2, \text{succ } N3)}$$

**Figure 3:** The sum of natural numbers

1. Prove that the sum of even numbers is even, e.g.:  
 $\text{even}(N1) \wedge \text{even}(N2) \wedge \text{plus}(N1, N2, N3) \Rightarrow \text{even}(N3)$

