Assignment 10 - due June 13th

Send answers to dcc030ufmg@gmail.com.

Exercise 1 (Exercise 4.53)**.** Prove that

$$\mathrm{mc}(G) \leq \frac{n}{4} \min_{\mathbf{v} \perp \mathbf{1}} \lambda_n(\mathbf{L} + \mathbf{diag}(\mathbf{v})),$$

where $\mathrm{mc}(G)$ stands for the size of a maxcut in G.

Exercise 2. Consider the proof of the fact that

$$\Phi < \sqrt{2\Delta\lambda_2},$$

where a chain of 10 equalities and inequalities was shown. Explain why each one of them is true.

Exercise 3. Recall the definition of the normalized Laplacian of a graph G:

$$\mathbf{Q} = \mathbf{I} - \mathbf{D}^{-1/2}\mathbf{A}\mathbf{D}^{-1/2}.$$

Show that its largest eigenvalue, μ, satisfies $\mu \leq 2$. If G is connected, show that equality holds if and only if G is bipartite.