Assignment 11 - due July 4th

Send answers to `dcc030ufmg@gmail.com`.

Exercise 1. Let $\mathbb{F} = \mathbb{F}_q$. Assume there is a subset $B \subseteq \mathbb{F}^N$ so that, for each point $\mathbf{x} \notin B$, there is a line $L_{\mathbf{x}} = \{\mathbf{x} + t\mathbf{w} : t \in \mathbb{F}\}$ so that $|L_{\mathbf{x}} \cap B| = q - 1$. Prove that

$$|B| \geq \binom{n+q-2}{n}.$$

Exercise 2. Let G be a bipartite graph, classes U and V, and assume each class has n vertices. To each edge $uv \in E(G)$, consider the variable x_{uv}. Define the square $n \times n$ matrix $\mathbf{B}$, whose rows are indexed by vertices in U and columns by vertices in V, with $\mathbf{B}_{u,v} = x_{uv}$ if $uv \in E$, and 0 otherwise. Show that $\det \mathbf{B}$ is not identically equal to 0 if and only if G has a perfect matching.

Exercise 3. Suppose there are hyperplanes $H_1, H_2, ..., H_m$ in $\mathbb{R}^n$ that do not pass through the origin, but otherwise cover all $2^n - 1$ vertices of the hypercube $\{0,1\}^n$. Prove that $m \geq n$.