

Assignment 2 - due March 18th

Send answers to dcc030ufmg@gmail.com.

Exercise 1 (1.31 in the notes). Find the exponential generating function for the number of labelled graphs on n vertices such that each connected component is a regular graph of valency 2?

Exercise 2 (1.40 in the notes). For even n (only!), let e_n and o_n stand respectively for the number of permutations with all cycle of even and odd length. Let $E(x)$ and $O(x)$ be their exponential generating functions, and $P(x)$, again, the exponential generating function for all permutations (but recall, n is even!). Our goal is to show that $e_n = o_n$.

- (a) Verify that $P(x) = (1 - x^2)^{-1}$.
- (b) Prove that $E(x) = (1 - x^2)^{-1/2}$.
- (c) Argue that $P(x) = E(x) \cdot O(x)$. Conclude that $O(x) = E(x)$, and thus $e_n = o_n$.
- (d) Find a formula for e_n .
- (e) Try to find a bijective proof of the formula in (d), comparing a permutation with cycles of even length with 2 distinct partitions of the set into subsets of size 2.