Assignment 3 - due March 27th

Send answers to `dcc030ufmg@gmail.com`.

Exercise 1 (2.12 in the notes)**.** Prove that two symmetric matrices $\mathbf{M}$ and $\mathbf{N}$ commute if and only if they can be simultaneously diagonalized by the same set of orthonormal eigenvectors. Is it true that if $\mathbf{M}$ and $\mathbf{N}$ commute, then there is always a polynomial p so that $p(\mathbf{M}) = \mathbf{N}$? Characterize what else you need to observe to guarantee that such polynomial exists.
(If you want to do this exercise, you will have to show that if $\mathbf{M}$ and $\mathbf{N}$ commute, then each eigenspace of $\mathbf{M}$ is $\mathbf{N}$-invariant and vice versa. Then apply the result seen in class. However you are allowed to research this result if you want.)

Exercise 2 (2.28 in the notes)**.** Assume $G = (V, E)$ is a k-regular graph which contains a subset of vertices $U \subseteq V$ satisfying the following properties:

(a) No two vertices in U are neighbours.

(b) Any vertex in $V \backslash U$ contains exactly one neighbour in U.

Prove that if such U exists, then -1 is an eigenvalue of the graph.