Assignment 5 - due April 11th

Send answers to dcc030ufmg@gmail.com.

Exercise 1 (3.9 in the notes)**.** Let $\mathcal{P}_{ab}$ be the set of all paths from a to b. Prove that

$$\sqrt{\phi_{G\setminus a}(y)\phi_{G\setminus b}(y) - \phi_G(y)\phi_{G\setminus ab}(y)} = \sum_{P\in\mathcal{P}_{ab}} \phi_{G\setminus P}(y).$$

Exercise 2 (3.12 in the notes)**.** Let $w(x)$ be the generating function whose coefficient of x^k count the total amount of all walks in the graph of length k. Show that

$$w(x) = \frac{1}{x}\left(\frac{(-1)^n\phi_{\overline{G}}(-1-x^{-1})}{\phi_G(x^{-1})} - 1\right).$$

Exercise 3 (For this exercise, read section 2.5 again)**.** The goal of this exercise is to show that there is no strongly regular graph with parameters $(28, 9, 0, 4)$.

(a) If it exists, what would be its eigenvalues and corresponding multiplicities?

Now assume there is a strongly regular graph with parameters (n, k, a, c). Let $\delta = a - c$. Verify that

(b) $(x^2 - \delta x - k + c)(x\mathbf{I} - \mathbf{A})^{-1} = \mathbf{A} + (x - \delta)\mathbf{I} + \frac{c}{x-k}\mathbf{J}$.

Assume now D is an independent set of a strongly regular graph G (meaning, no two vertices in D are adjacent). Using (b) above and Theorem 3.8 from the notes...

(c) Derive a formula for $\phi_{G\setminus D}(x)$ in terms of x, the eigenvalues of G and their multiplicities.

Conclude the exercise:

(d) Prove that there is no strongly regular graph with parameters $(28, 9, 0, 4)$.