Assignment 6 - due May 6th

Send answers to `dcc030ufmg@gmail.com`.

Choose 5 exercises! (Well... but why not try all?). All exercises below are from the notes. Find them there to get context and more hints.

Exercise 1. Prove that

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \mathrm{e}^{-x^2/2} \mu_{K_n}(x) \mu_{K_m}(x) \mathrm{d}x = \begin{cases} m!, & \text{if } m = n; \\ 0, & \text{otherwise.} \end{cases}$$

Exercise 2. Prove that the zeros of $\mu_{G \backslash u}$ interlace those of μ_G. If G is connected, prove that the largest zero of μ_G is simple, and strictly larger than that of $\mu_{G \backslash u}$.

Exercise 3. Show (again) that the largest eigenvalue of a non-negative matrix is upper bounded by its largest row sum.

Exercise 4. Extend the result above to argue that the largest eigenvalue of a non-negative matrix $\mathbf{M}$ is upper bounded by the largest row sum of $\mathbf{DMD}^{-1}$ for any positive diagonal matrix $\mathbf{D}$.

Exercise 5. Let T_Δ be a tree so that all vertices have degree $\Delta > 2$ or 1. Prove that its largest eigenvalue is upper bounded by $2\sqrt{\Delta - 1}$.

Exercise 6. Argue that any tree of maximum degree $\Delta > 1$ has its largest eigenvalue small or equal than $2\sqrt{\Delta - 1}$.

Exercise 7. Let G be a graph with $\Delta(G) > 1$. Show that the largest root λ of $\mu_G(x)$ satisfies

$$\sqrt{\Delta(G)} \leq \lambda \leq 2\sqrt{\Delta(G) - 1}.$$

Exercise 8. Assume G is a graph with the property that every cycle of G contains at least one edge that belongs to no other cycle. Show how to compute μ_G efficiently.

Exercise 9. Prove by induction, using the edge deletion/contraction definition, that

$$T_G(x, y) = \sum_{A \subseteq E} (x-1)^{r(E)-r(A)} (y-1)^{n(A)}$$

Exercise 10. Prove that the Tutte polynomial is reconstructible from the deck. You can use results seen in class.