

Assignment 8 - due May 23rd

Send answers to dcc030ufmg@gmail.com.

Choose 2.

Exercise 1 (4.17 in the notes). Show that if $\theta_2 > 0$, then

$$\chi(G) \geq 1 - \frac{\theta_{n-\chi+1}}{\theta_2}.$$

Exercise 2 (4.34 in the notes). Assume G is connected, on n vertices, and let λ_2 be its second smallest Laplacian eigenvalue. Show that

$$\lambda_2 = \min_{\mathbf{v} \neq \alpha \mathbf{1}} \frac{n \sum_{ab \in E(G)} (\mathbf{v}_a - \mathbf{v}_b)^2}{\sum_{a < b} (\mathbf{v}_a - \mathbf{v}_b)^2}.$$

(The minimum is being taken over all vectors which are just not constant.)

Exercise 3. Let G be a graph on n vertices, δ its smallest degree, α its coclique number, χ its chromatic number. Let \mathbf{L} be its Laplacian matrix, and λ its largest eigenvalue. Prove that

$$\alpha \leq n \frac{\lambda - \delta}{\lambda}.$$