Assignment 9 - due May 30th

Send answers to dcc030ufmg@gmail.com.

Let G be a graph, $\mathbf{L}(G)$ its Laplacian. We assume its eigenvalues are $\lambda_1 \leq \lambda_2 \leq ... \leq \lambda_n$.

Exercise 1. Given a connected electrical network, the effective resistance between any pair of vertices is defined as

$$R_{\text{eff}}(a,b) = (\mathbf{e}_a - \mathbf{e}_b)^T \mathbf{L}^+ (\mathbf{e}_a - \mathbf{e}_b).$$

Show that the sum of the effective resistances between all pairs of vertices is equal to

$$n \sum_{i=2}^{n} \frac{1}{\lambda_i}.$$

Exercise 2. We saw that for all trees on more than 2 vertices, $\lambda_2 \leq 1$. Prove that equality holds if and only if the tree is a star.

Exercise 3. Let G be a graph, and $e \in E(G)$. Prove that

$$\lambda_2(G \backslash e) \leq \lambda_2(G) \leq \lambda_2(G \backslash e) + 2.$$

Show that equality holds in the second bound if and only if G is complete.

Exercise 4. Let G be a graph on n vertices, with diameter d. Show that $\lambda_2 \geq 1/nd$.