

Algebraic Combinatorics

Prerequisites: intermediate knowledge of linear algebra, basics of graph theory, combinatorics of counting and calculus. Comfort level of reading/listening English.

Main topics: Generating functions. Spectrum of a graph. Polynomials in combinatorics.

Syllabus: Power series, generating functions, exponential generating functions, examples. Matching polynomials. Symmetric matrices, and the Perron-Frobenius Theorem. Adjacency matrix of a graph. Walk generating function of a graph. Rayleigh quotients, and Cauchy's interlacing. Combinatorial eigenvalue bounds. Matrix-tree theorem. Cheeger's inequality. Spectral Algorithms. Random walks and expanders. Examples of rank arguments in combinatorics. DeMillo–Lipton–Schwartz–Zippel Lemma and the Kakeya problem. Tutte matrix of a graph and matchings. Combinatorial Nullstellensatz and applications.

For whom is this course intended: first and foremost, you have to like mathematics, specially algebra and combinatorics. Be warned that you will be required to write proofs, and the pace will be fast. I expect that you spend 6 hours/week with it outside of class.

English: this course will be taught in English for one sole good reason. Every useful thing that you might be able to do with what you learn here will have to be done in English. Lectures will be given in English, but I will *not* require that you speak English when asking questions. However, assignments will have to be handed in in English.

Evaluation: 100 points on assignments. Everything will have to be LaTeXed.

Lectures: Tuesdays and Thursdays, 9h25 - 11h05.

References:

- J. H. van Lint and R. M. Wilson. *A course in Combinatorics*. Second Edition. Cambridge.
- M. Aigner. *A Course in Enumeration*. Springer.
- C. Godsil. *Algebraic Combinatorics*. Chapman Hall.
- C. Godsil and G. Royle. *Algebraic Graph Theory*. Springer.
- A. Brouwer and W. Haemers. *Spectra of Graphs*. Springer.
- F. R. K. Chung. *Spectral Graph Theory*. CBMS.
- R. Stanley. *Algebraic Combinatorics: Walks, Trees, Tableaux, and More*. Second Edition. Springer.
- S. Jukna. *Extremal Combinatorics*. Second Edition. Springer.