Assignment 2 - due August 18th

Send pdf with answers to `dcc030ufmg@gmail.com`. They must be typed up in latex.

This assignment corresponds largely to (some sections of) Chapter 1 and to Sections 2.5 and 2.6 of the Graph Spectra and Quantum Walks book (see link on the webpage).

1st part

Answer the following questions either in a itemized manner, or by writing a short summary of the related theory.

- Read section 2.5 in detail. Explain in your own words why one block of H_{xy} if equal to the adjacency matrix of the graph, namely, explain why H_{xy} applied at the state $f_{\{a\}}$ is equal to the state $\sum f_{\{j\}}$, where the sum is over the vertices neighbouring a. You should compute H_{xy} for the graph P_3 and use it as an explicit example of what is happening.
- Now read the beginning of Chapter 1. Note that because $H = H_{xy}$ is a block matrix, then $\exp(\mathrm{i}tH)$ will also be a block matrix, one of its blocks equal to $\exp(\mathrm{i}tA)$. Explain how the spectral decomposition of A will be useful to compute the spectral decomposition of $\exp(\mathrm{i}tA)$.
- Explain what are perfect state transfer, instantaneous uniform mixing, and average mixing matrix (check later chapter).

2nd part

Answer the following short exercises.

Exercise 1. Show that if A and B are square commuting matrices, then $\exp(A + B) = \exp(A)\exp(B)$. Provide an example of two matrices A and B that do not commute and so that the equality above does not hold for them.

Exercise 2. Let A be the adjacency matrix of a graph G. If k is a nonnegative integer, and a and b are vertices of G what does the number $(A^k)_{ab}$ count?

Exercise 3. Show that a connected graph G is regular (that is, all its vertices have the same degree) if and only if $A(G)$ commutes with the all 1s matrix J.

Exercise 4. Let $\overline{X}$ be the complement of a regular graph X on n vertices, that is, $\overline{A} = A(\overline{X}) = J - I - A(X)$. Show that at times t which are integers multiples of $2\pi/n$, $\exp(\mathrm{i}t\overline{A})$ and $\exp(\mathrm{i}tA)$ are very much alike.

Exercise 5. Show that in the complete bipartite graph $K_{2,n}$ we have perfect state transfer between the vertices of degree n.

Exercise 6. Show that uniform mixing occurs in K_3 and in C_4 (note that C_4 is $P_2 \square P_2$).