Assignment 7 - due September 15th

Send pdf with answers to dcc030ufmg@gmail.com. They must be typed up in latex.

This assignment is about Chapter 10 of the book Graph Spectra and Quantum Walks, specially sections 10.5 - 10.9.

1st part

- The goal this week is to understand the proof that there is no state transfer in trees according to the quantum walk model defined with the Laplacian matrix.
- Write a complete proof of the following characterization theorem: Let G be a graph, with Laplacian matrix L, and vertices a and b. There is perfect state transfer between a and b at time t relative to L if and only if the following hold (1) a and b are strongly cospectral (relative to Laplacian) (2) eigenvalues in the support of a and b are integers (3) if g is the gcd of the eigenvalues, then λ/g is even if and only if $E_\lambda e_a = E_\lambda e_b$.
- Read sections 10.8 and 10.9 in detail.

2nd part

Exercise 1. This exercise aims to provide an alternative proof to Theorem 10.9.1. Let G be a tree. Assume $Lv = \lambda v$, with λ integer and v integer, so that the gcd of its entries is equal to 1.

(a) Recall that $L = BB^T$, where B is the incidence matrix of any orientation of L. Show that all entries of v are congruent to the same thing modulo λ (hint: prune leaves).

(b) Conclude that if $v_a = -v_b$ for some a and b, then $\lambda = 1$ or $\lambda = 2$.

(c) Use this to argue that, if there is perfect state transfer, then there can be only one eigenvalue λ so that $E_\lambda e_a = -E_\lambda e_b$.

Exercise 2. Show that there is no perfect state transfer in trees with more than 3 vertices according to the quantum walk model defined on the adjacency matrix.

The previous exercise is actually an open question :)
You can assume eigenvalues are integers if you want. Write out some ideas. Anything could be useful (to me).

Exercise 3. This week things are lighter because I want you to start reading chapter 14 (specially 14.2 - 14.5).