Assignment 9 - due October 6th

Send pdf with answers to dcc030ufmg@gmail.com. They must be typed up in latex.

This assignment is about Chapter 3 of the book Graph Spectra and Quantum Walks.

1st part

- This is a short chapter. No need to write a summary. Read it carefully and make sure to understand what is going on.

2nd part

Solve **all six exercises** in the end of the chapter.

Additionally, solve the following two exercises:

Exercise 1. Find a graph that admits a non-trivial equitable partition that contains at least one class with at least two vertices so that no automorphism of the graph maps one of them to the other.

Exercise 2. Let M be the weighted adjacency matrix of a weighted path. Assume M has 0 diagonal. As we have seen, M is diagonally similar to any other tridiagonal matrix that preserves the product $M_{i,i+1}M_{i+1,i}$ for all i. Let R be the permutation matrix with the all ones anti-diagonal.

(a) Assume M has ones in its lower subdiagonal, and integers on the upper subdiagonal, all the way to the centre of the matrix. Also, assume $RMR = M$. Describe a graph that admits M as the quotient matrix of an equitable partition.

(b) Find an example of a matrix M satisfying the properties described in (a) and another tridiagonal matrix N so that $M_{i,i+1}M_{i+1,i} = N_{i,i+1}N_{i+1,i}$ for all i, but so that N is the quotient matrix of and equitable partition of a smaller graph.