Better SMT proofs for certifying compliance

(or what a bunch of people achieve in 1.5 years writing from scratch a proof module for an state-of-the-art SMT solver)

Haniel Barbosa

FACC 2021

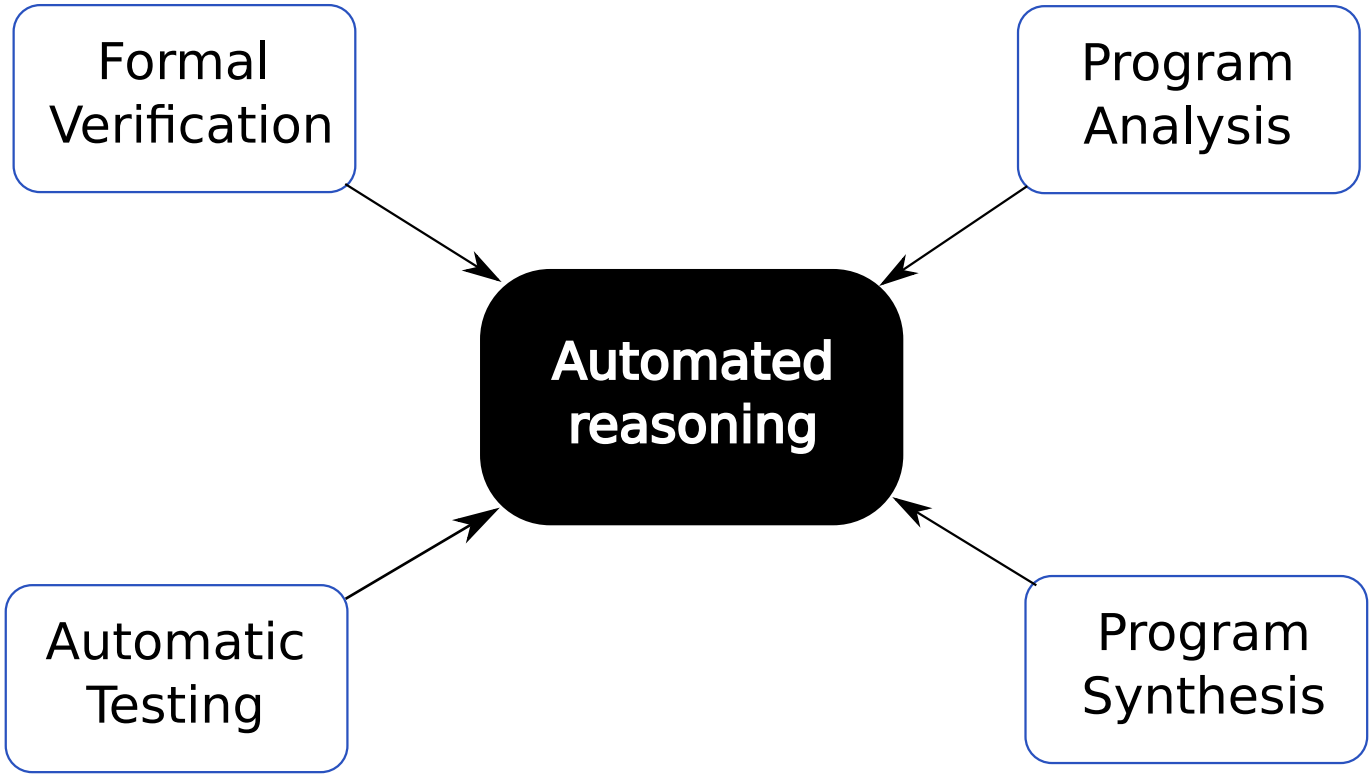
2021–07–19, The Internet

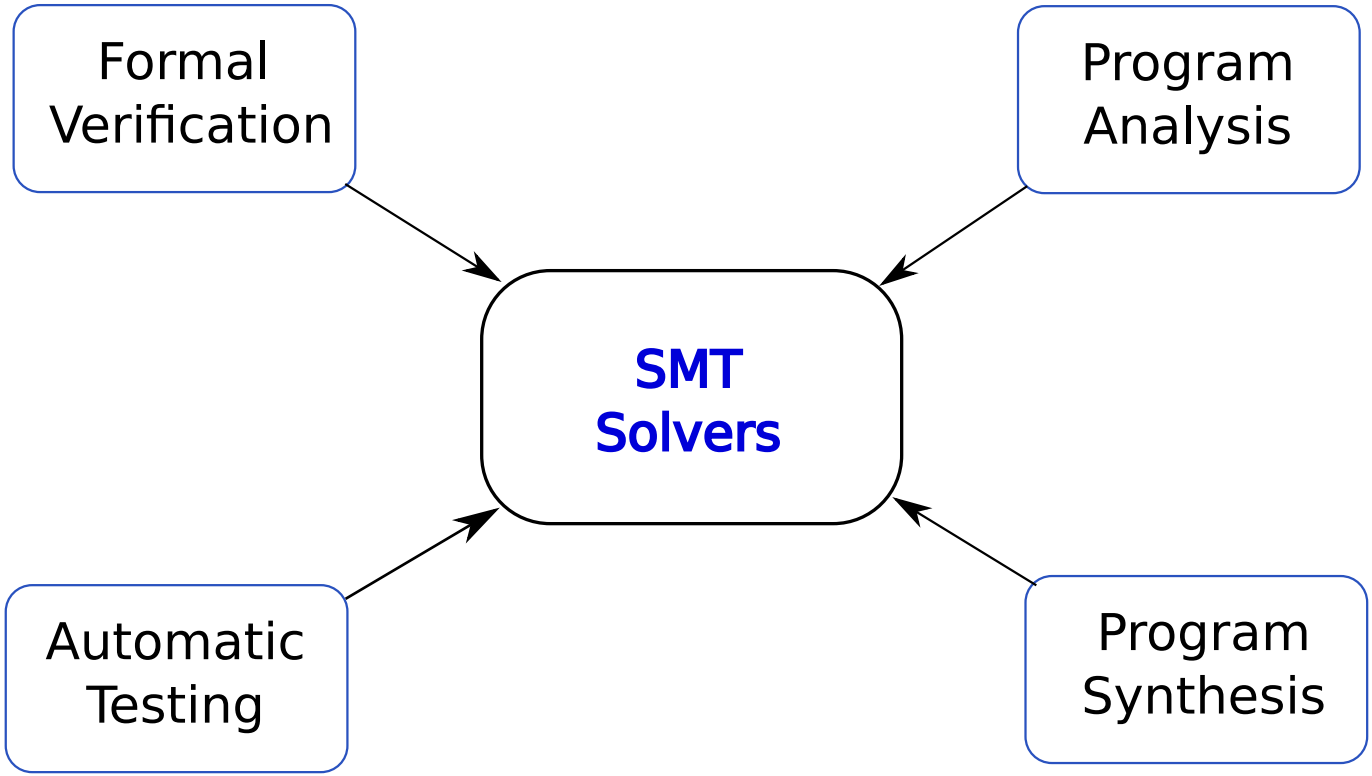
Formal Verification

Program Analysis

Automatic Testing

Program Synthesis


Formal Verification
Program Analysis
Automated reasoning
Automatic Testing
Program Synthesis


Formal
Verification
Program
Analysis
SMT
Solvers
Automatic
Testing
Program
Synthesis

SMT solvers can be hard to trust

In many settings one cannot just assume the solver is correct. But:

- Despite the best effort of developers, bugs remain
- Every year SMT-COMP has numerous disagreements between solvers
- Fuzzing tools often find bugs in solvers

Why don't we just certify/qualify the solvers?

- Large, complex code bases are too costly to certify
- A (simpler) certified system can be too slow [FBL18; Fle19]
- Certifying/qualifying a system freezes it, potentially blocking improvements
  - Working around adding new features slow and costly [BD18]

For your consideration: proofs!

- Proofs are a justification of the logical reasoning the solver has performed to find a solution
- A proof can be checked *independently*
  - And generally more efficiently than solving the problem
- Confidence in results is decoupled from the solver's implementation

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- Confidence in results is decoupled from the solver's implementation
- Why are proofs not common place in compliance involving solvers?

Challenges for SMT proofs

- Collecting and storing proofs efficiently

  many attempts, no panacea

  [SZS04; KBT+16; HBR+15; Mos08; MB08; Sch13; KV13; WDF+09; BODF09]

- Proofs for sophisticated preprocessing and rewriting techniques

  initial progress but many challenges remain

  [BBFF20]

- Proofs for some decision procedures (e.g. CAD, strings)

  open

- Standardizing a proof format

  open

- Scalable, trustworthy checking

  many attempts, no panacea

  [BBP13; SOR+13; EMT+17; BBFF20; SFD21]

The cvc5 team's plan

- CVC4's old proof module struggled with many of those challenges
- Since early 2020, the cvc5 team has been redesigning its proof module *from scratch*
  - Haniel Barbosa, Andrew Reynolds, Alex Ozdemir, Hanna Lachnitt, Scott Viteri, Diego Camargos, Gereon Kremer, Aina Niemetz, Mathias Preiner, Andres Nötzli, Yoni Zohar, Cesare Tinelli, Clark Barrett

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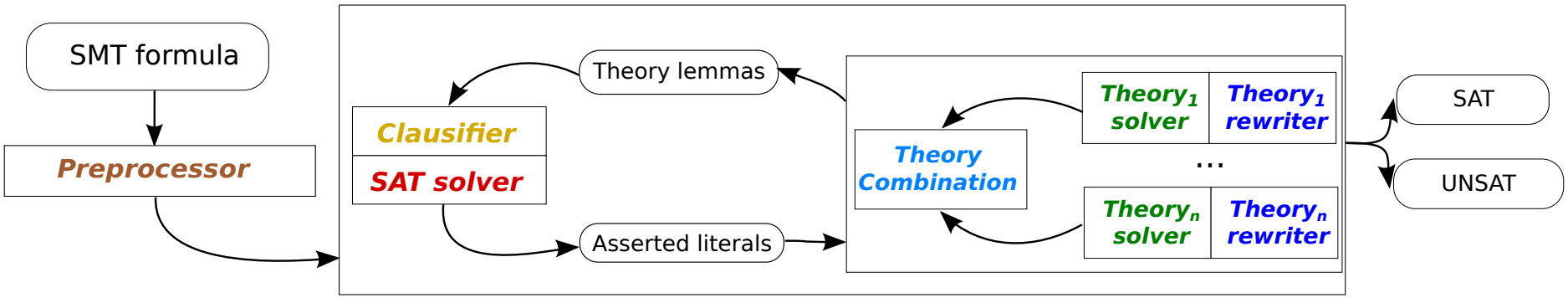
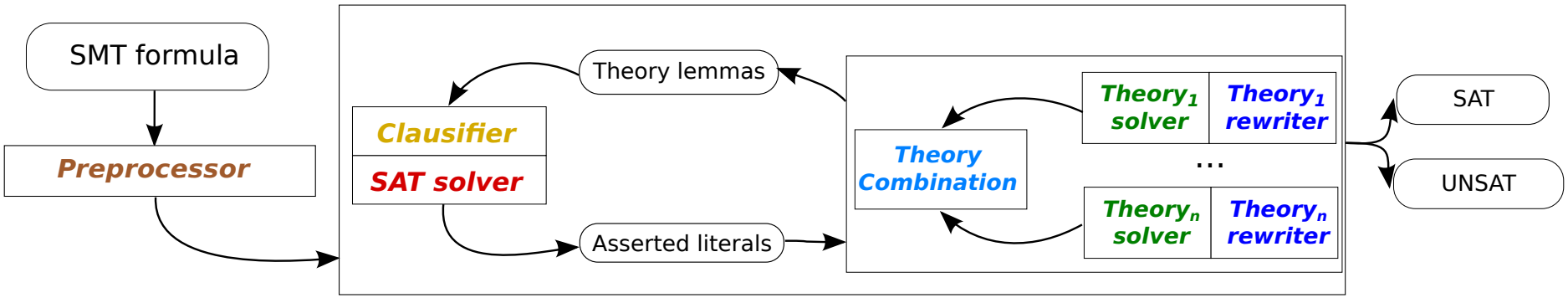
  - Allow custom eager/lazy generation of proofs
  - Support different proof formats (and different external proof checkers)

How an SMT solver works: CDCL($\mathcal{T}$) calculus



How an SMT solver works: CDCL($\mathcal{T}$) calculus



- ▷ Preprocessor simplifies formula globally

$x \simeq t \wedge F[x] \;\mapsto\; F[t]$ $\qquad$ $F[(ite\ P\ t_1\ t_2)] \;\mapsto\; F[t'] \wedge t' \simeq (ite\ P\ t_1\ t_2)$

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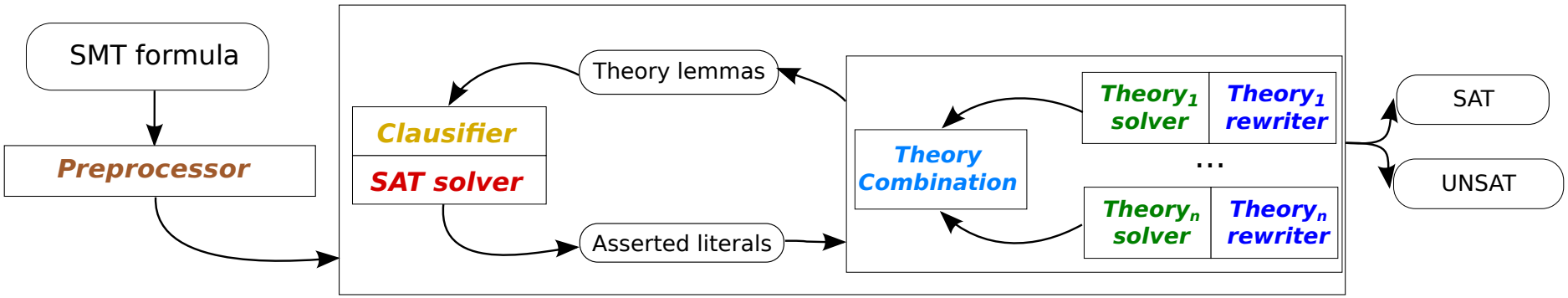

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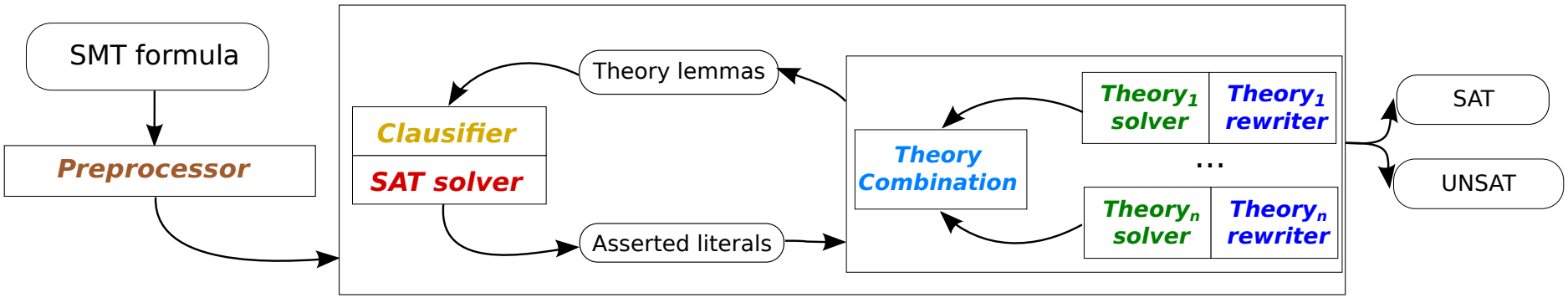

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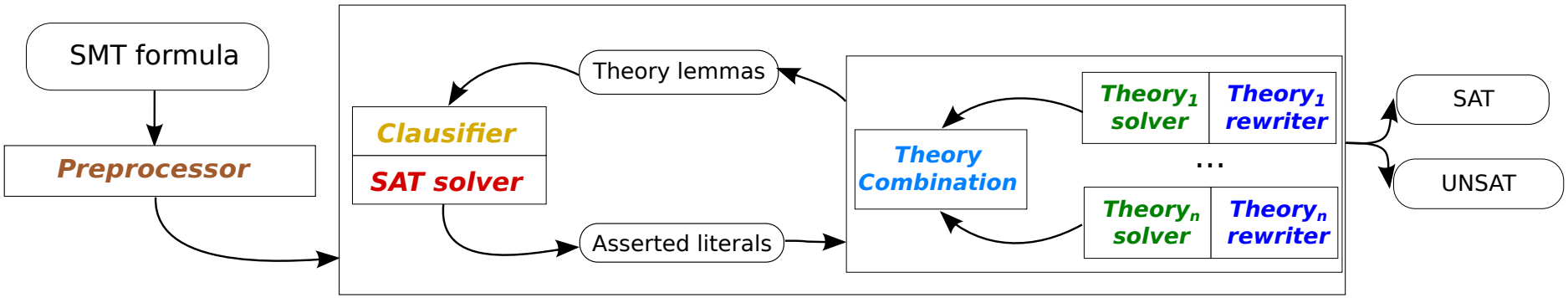

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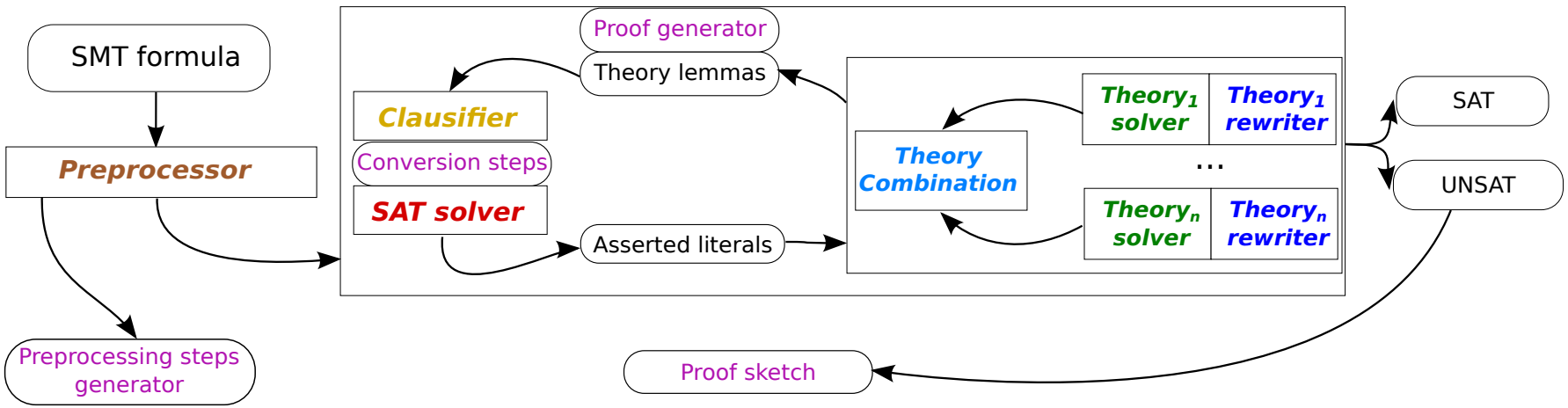
- ▷ Clausifier converts to Conjunctive Normal Form (CNF)
- ▷ SAT solver asserts literals that must hold based on Boolean abstraction
- ▷ Theory solvers check consistency in the theory
- ▷ Theory rewriters simplify terms of their theories

  $x + 0 \to x$ $\qquad$ $\mathsf{a} \not\simeq \mathsf{a} \to \bot$ $\qquad$ $(\text{str.replace}\ x\ (\text{str.++}\ x\ x)\ y) \to x$

Proof module architecture



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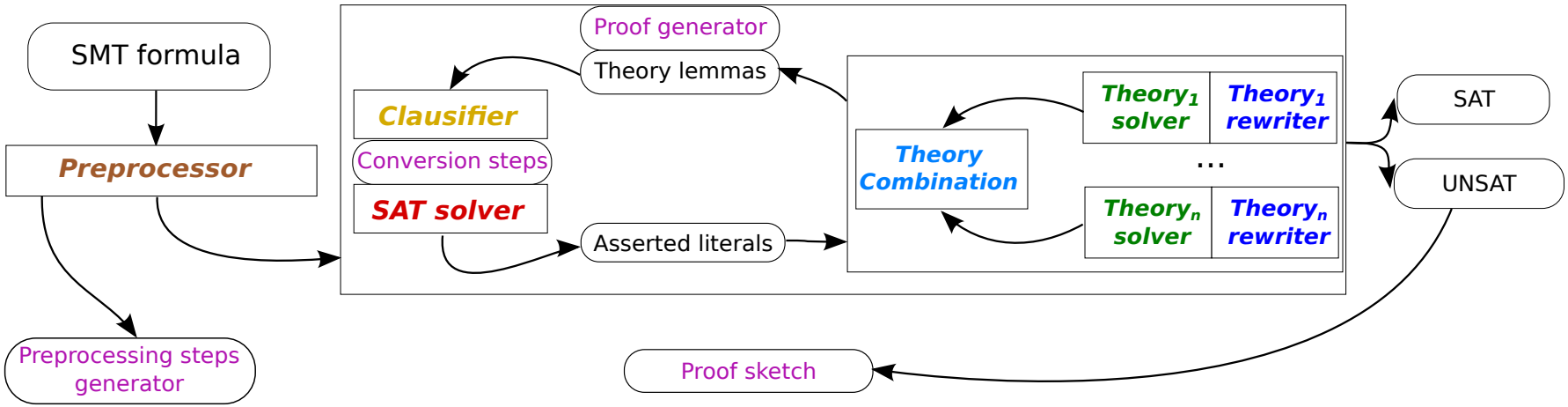


- Proof sketch contains a *resolution* proof of unsatisfiability of CNF
  - Leafs are theory lemmas and preprocessed assertions

$$\frac{A \vee \ell \qquad B \vee \bar{\ell}}{A \vee B}$$

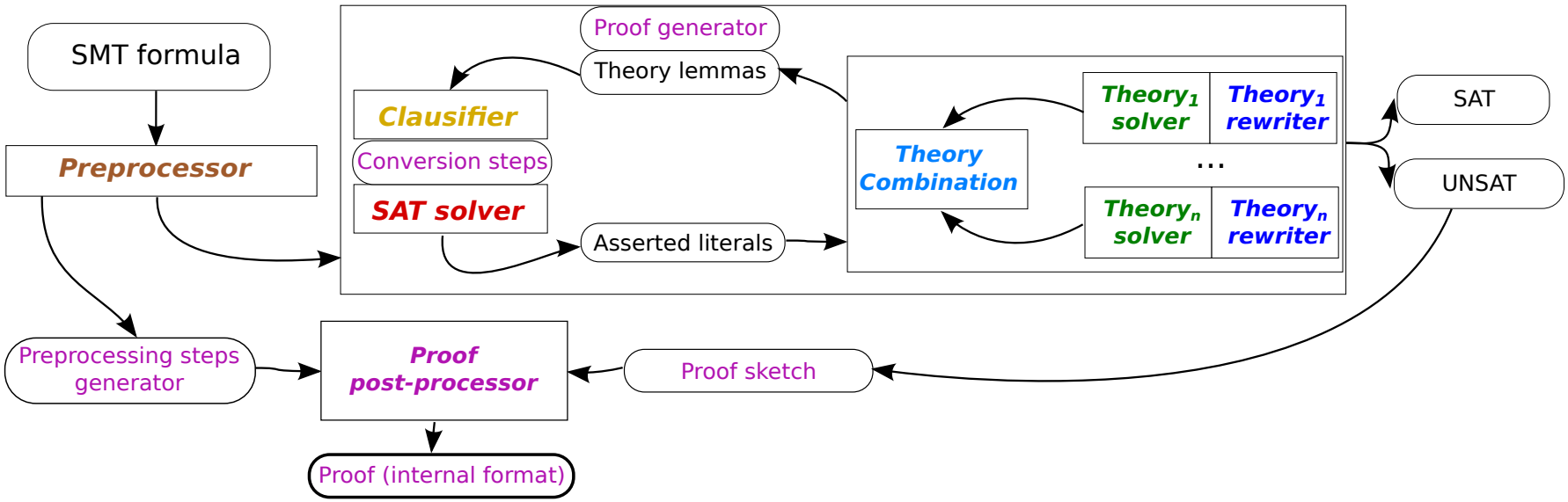
$$\neg(a \simeq b) \vee f(a) \simeq f(b)$$
$$\neg(y > 1) \vee \neg(x < 1) \vee y > x$$

$$\frac{\varphi_1 \wedge \cdots \wedge \varphi_n}{\varphi_i}$$

$$\neg(\varphi_1 \wedge \cdots \wedge \varphi_n) \vee \varphi_i$$

- Generators are hooks to proof-producing modules in preprocessor/theory reasoners

Proof module architecture



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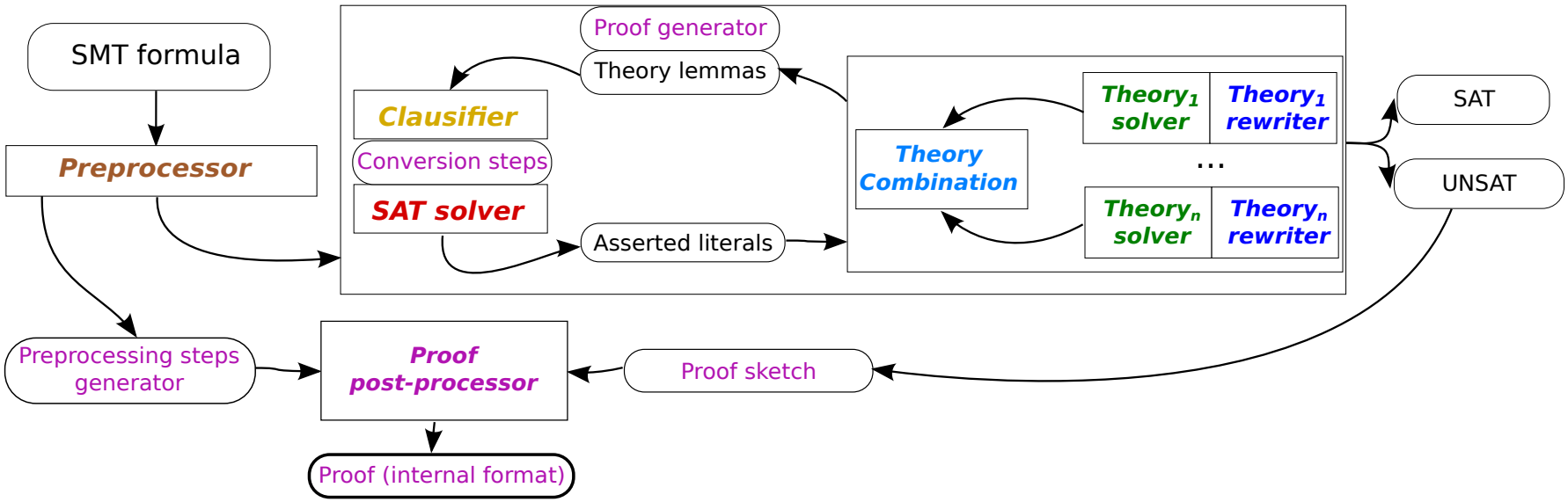
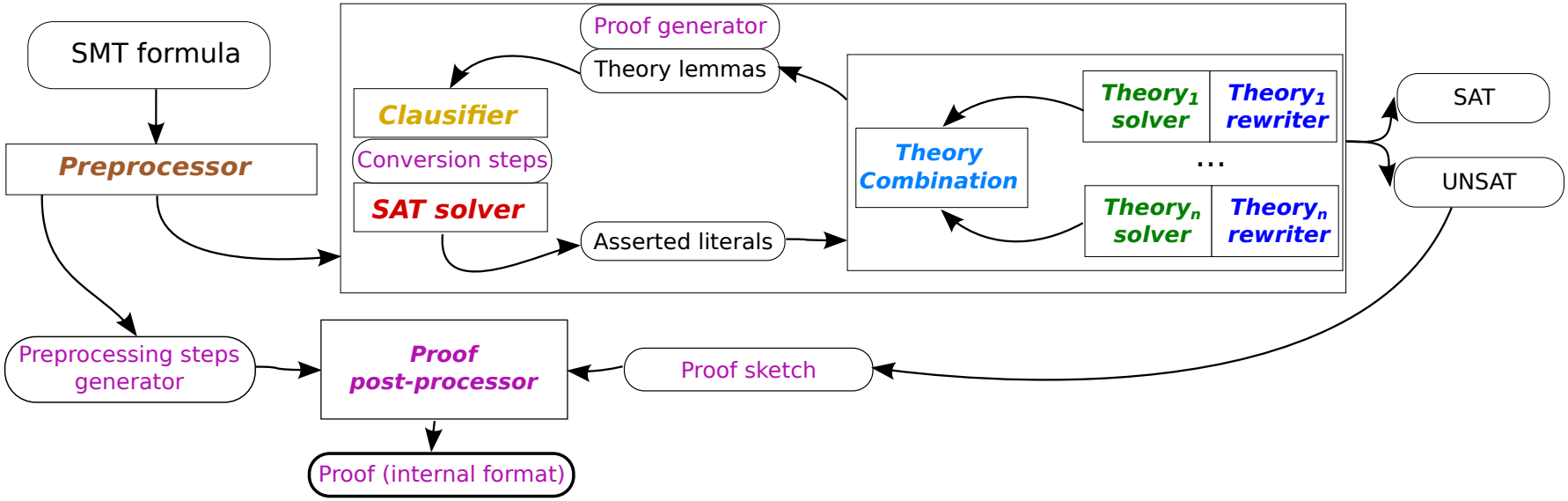


- ▷ Post-processor invokes proof generators of leaves in sketch
- ▷ Also (optionally) expands coarse-grained steps into fine-grained ones

$$\frac{a \simeq 0 \qquad b \simeq 1}{(a > b \wedge F) \simeq \bot}\ \text{MACRO}$$

Proof module architecture



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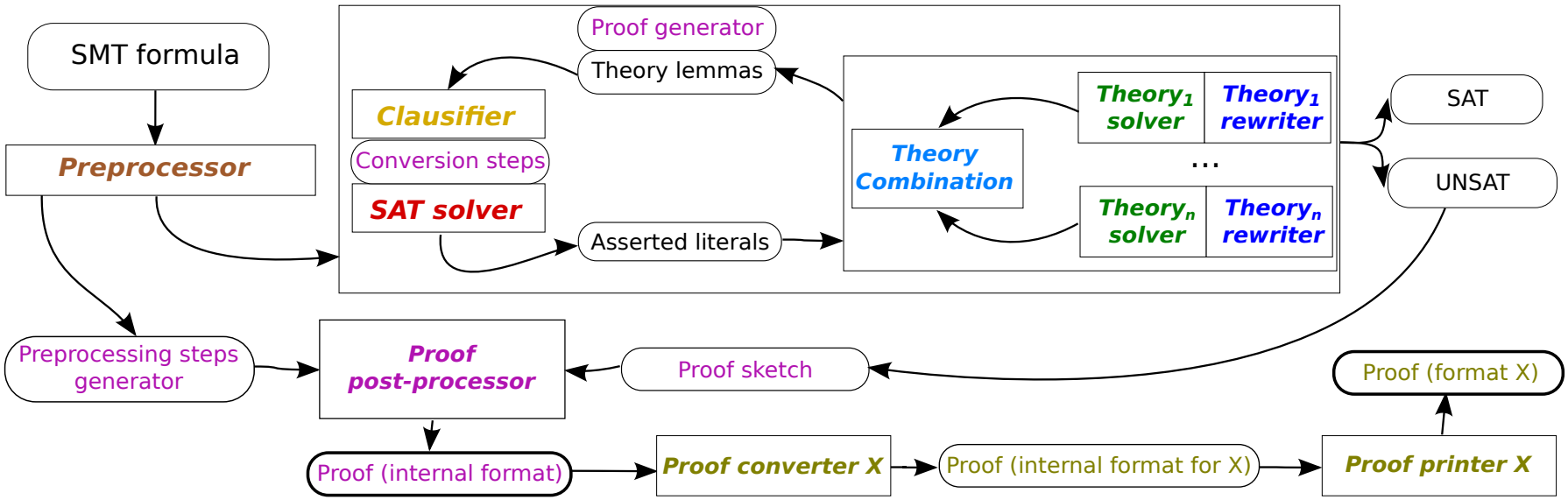
$$\dfrac{\dfrac{a \simeq 0 \qquad b \simeq 1}{(a > b \wedge F) \simeq (0 > 1 \wedge F)}\ \text{SUBS} \qquad \dfrac{}{(0 > 1 \wedge F) \simeq \bot}\ \text{RW}}{(a > b \wedge F) \simeq \bot}$$

Proof module architecture



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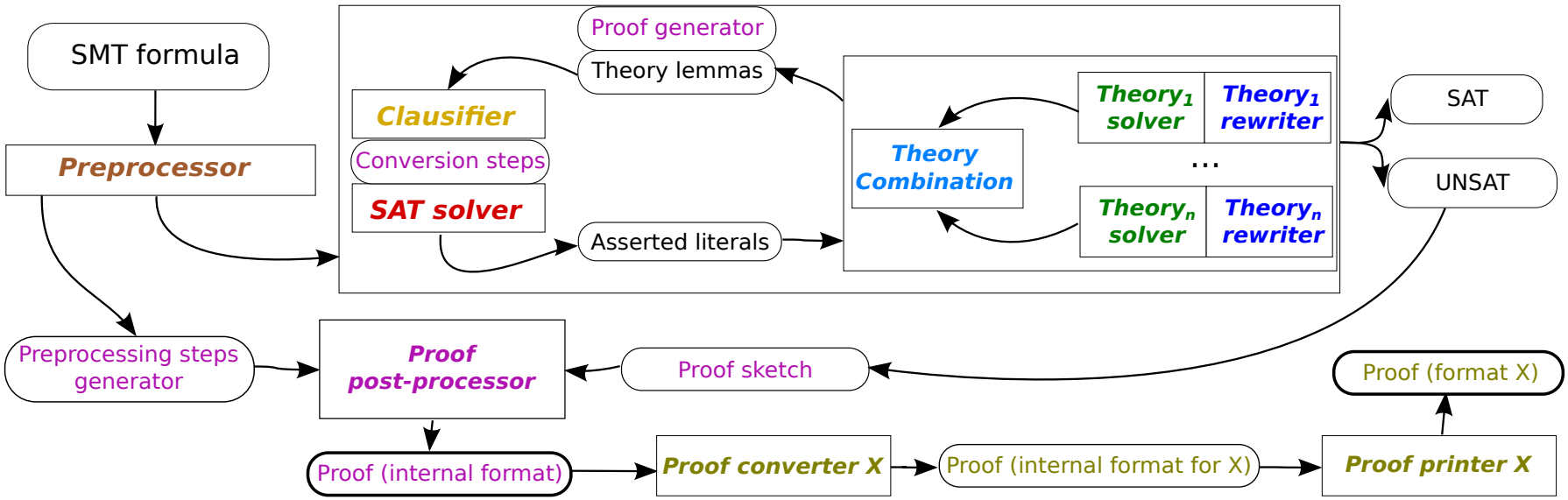

- Internal proof can be converted and printed to different proof formats
  - Convert the AST of internal proofs based on X
  - Pretty printer that proof in format X
- No internal proof checking for X, since it'll be checked externally

Demo!

Consider the following unsatisfiable SMT problem

```
(set-logic QF_UF)
(declare-sort U 0)
(declare-const p1 Bool)
(declare-const p2 Bool)
(declare-const p3 Bool)
(declare-const a U)
(declare-const b U)
(declare-fun f (U) U)

(assert (= a b))
(assert (and p1 true))
(assert (or (not p1) (and p2 p3)))
(assert (or (not p3) (not (= (f a) (f b)))))
```

A particular challenge has been String solving

- Preprocessing
- Clausification
- SAT solving
- UF theory solver
- Linear Arithmetic solver
- Theory combination
- Quantifier instantiation
- Rewriting
  - Including complex string methods [RNBT19]
- Strings theory solver
  - Core calculus [LRT+14]
  - Extended function reductions [RWB+17]
  - Regular expression unfolding

Anecdotes

- Internal proof checker is highly valuable for development
- Error localization for proofs is important
- Formalization of proof rules uncovers existing issues
- Performance issues
  - In a few cases, proof checker indicated it could prove something stronger
- Soundness issues
  - Cannot write proper proof checker if the reasoning of the solver is wrong
- Proofs are also valuable for debugging
  - Soundness bug reported, proofs used to easily isolate the incorrect rewrite

Ongoing work

- Conversions to different proof formats
  - LFSC: custom checker via LFSC's type checking
  - Lean: custom checker via Lean4's type checking
  - Alethe
    - proof reconstruction in Isabelle/HOL via Sledghammer
    - proof reconstruction in Coq via SMTCoq
    - custom checker in Rust
  - Dot: proof visualization
- Fine-grained reconstruction of theory rewrite steps
  - Simple rewrite rules specified in DSL
  - Compilation into unification goals in cvc5
  - Elaboration of complex rewrite steps in terms of simple ones from DSL
- Incorporating DRAT proofs for BV lemmas from black-box SAT solvers

Conclusion

- For 1.5 years the cvc5 team has been hard at work to produce better proofs
- Fine-grained proofs are now available for most of cvc5's reasoning
  - Proofs for the strings solver has been a special milestone
- Multiple proof formats are supported
- Integration into multiple proof checkers are ongoing
  - Including the formalization of new calculi in Lean, LFSC, Isabelle/HOL
- We expect the high-quality proofs produced by cvc5 to facilitate automated compliance

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