

**HW 1: DFAs and NFAs***Instructor: Haniel Barbosa, TA: Shantanu Agarwal**Due date: Jan 31, 2018*

1. (a) Design and draw a DFA which reads in a non-negative integer written in binary, least significant bit first, and accepts exactly when the input is divisible by four. (There is no sign bit in the representation.) Specify what  $Q$ ,  $\Sigma$ ,  $\delta$ ,  $q_0$ , and  $F$  are for your DFA.  
(b) Now suppose that before the least significant bit, there is a bit indicating the sign of the integer: 0 for a negative number, and 1 for a positive number. How would you modify your DFA to handle these signed integers?
2. Consider the set of all binary strings where the difference between the number of 0s and the number of 1s is even.
  - (a) Is this set a valid alphabet for a DFA? Why or why not?
  - (b) Design and draw a DFA with as few states as possible recognizing this set.
3. Let  $L$  be the language of all strings over the alphabet  $\Sigma = \{a, b\}$  which does not contain the substring  $aa$  and does not contain the substring  $bb$  and has length at least 1.
  - (a) Design and draw an NFA which recognizes  $L$  and uses only three states.
  - (b) Explain each step of the accepting computation taken by your NFA on the string  $baba$ .
  - (c) Can any DFA recognize this language with only three states? Give such a DFA or show why one cannot exist.
4. Let  $L$  denote a language. The reversal of  $L$ , denoted  $L^R$ , is the language  $\{x^R : x \in L\}$ , i.e. the set of all strings whose reversal is in  $L$ . For example, if  $L$  is the set of all strings of length at least two whose first two bits are zero, then  $L^R$  is the set of all strings of length at least two whose last two bits are zero.  
Suppose you are given a DFA  $M$  with alphabet  $\Sigma$  that recognizes  $L$ . How would you construct an NFA  $M^R$  that recognizes  $L^R$ ? (Your NFA can have  $\epsilon$ -transitions.) Prove that your construction is correct.