

HW 3: Context-free Languages

Instructor: Haniel Barbosa, TA: Shantanu Agarwal

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1. (9 points)

Give context-free grammars that generate the following languages. In all parts, the alphabet Σ is $\{0, 1\}$

- (a) $\{w \mid w \text{ starts and ends with the same symbol}\}$
- (b) $\{w \mid \text{the length of } w \text{ is odd}\}$
- (c) $\{w \mid w = w^R, \text{ that is, } w \text{ is a palindrome}\}$

2. (11 points)

Give a context-free grammar that generates the following language, with alphabet Σ is $\{0, 1, \#\}$:

$$\{x_1\#x_2\#\dots\#x_k \mid k \geq 1, \text{ each } x_i \in \{0, 1\}^*, \text{ and for some } i \text{ and } j, x_i = x_j^R\}$$

3. (10 points)

Define the *size* of a context-free grammar to be the total number of characters used in writing the rules of the grammar down (including nonterminals, terminals, $|$ and \rightarrow). For example, the one-rule grammar $A \rightarrow A1 \mid \epsilon$ has size six since it uses six characters.

Consider a grammar that generates *only* the string “manamana banana” and no other strings. Here the set of terminals is the set of small letters in the English alphabet and the whitespace character (denoted explicitly by \sqcup), i.e. it is the set $\{a, b, c, \dots, z, \sqcup\}$. The smallest CFG that generates only this string has size *sixteen*. Write this grammar.

4. (20 points)

Convert the CFG

$$\begin{aligned} R &\rightarrow XRX \mid S \\ S &\rightarrow aTb \mid bTa \\ T &\rightarrow XTX \mid X \mid \epsilon \\ X &\rightarrow a \mid b \end{aligned}$$

to an equivalent PDA following the procedure given in the proof of the theorem shown in class on the equivalence of CFGs and PDAs. Include all steps.

5. (30 points)

Let $\Sigma = \{0, 1, \#\}$. Design a context-free grammar that generates the language

$$L = \{x\#y \mid x, y \in \{0, 1\}^*, x \neq y\}$$

Explain why your grammar is correct.

Also, give the state-diagram of a PDA that recognizes L . (No need to prove that your PDA is correct.)

6. (20 points)

Let B be the language of all palindromes over $0, 1$ containing equal numbers of 0s and 1s. Show that B is not context-free.