CS:4330 Theory of Computation **Spring 2018**

HW 5: (Un)Decidability

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1. (15 points) Let $INFINITE_{\text{PDA}} = \{\langle M \rangle \mid M \text{ is a PDA and } \mathcal{L}(M) \text{ is an infinite language}\}$. Show that $INFINITE_{\text{PDA}}$ is decidable.

2. (15 points) Let $A = \{\langle R, S \rangle \mid R \text{ and } S \text{ are regular expressions and } \mathcal{L}(R) \subseteq \mathcal{L}(S)\}$. Show that A is decidable.

3. (15 points) Let $C = \{\langle G, x \rangle \mid G \text{ is a CFG and } x \text{ is a substring of some } y \in \mathcal{L}(G)\}$. Show that C is decidable. (Hint: An elegant solution to this problem uses the decider for E_{CFG}, the language encoding the emptiness problem for CFGs.)

4. (15 points) Show that the set of all infinite binary sequences is uncountable. An *infinite binary sequence* is an unending sequence of 0s and 1s.

5. (20 points) Show that $EQ_{DFA} = \{\langle A, B \rangle \mid A, B \text{ are DFAs, and } \mathcal{L}(A) = \mathcal{L}(B)\}$ is decidable by testing the two DFAs on all strings up to a certain size. Calculate a size that works.

6. (20 points) Let A be a Turing-recognizable language consisting of descriptions of Turing Machines, $\{\langle M_1 \rangle, \langle M_2 \rangle, \dots\}$, where every M_i is a decider. Show that some decidable language D is not decided by any decider M_i whose description appears in A.
(Hint: You may find it helpful to consider an enumerator for A.)