

HW 7: Reducibility and a bit of Time complexity*Instructor: Haniel Barbosa, TA: Shantanu Agarwal**Due date: April 9, 2018*

- (20 points) Show that a language A is decidable if and only if $A \leq_m 0^*1^*$, in which $L_1 \leq_m L_2$ denotes “ L_1 is mapping reducible to L_2 .”
- (15 points) Let $J = \{w \mid \text{either } w = 0x \text{ for some } x \in A_{\text{TM}}, \text{ or } w = 1y \text{ for some } y \in \overline{A_{\text{TM}}}\}$. Show that neither J nor \bar{J} is Turing-recognizable.
- (15 points) Consider the problem of determining whether a PDA accepts some string of the form $\{ww \mid w \in \{0,1\}^*\}$. Use the computation history method to show that this problem is undecidable.
- (30 points) Say that a variable A in CFG G is **necessary** if it appears in every derivation of some string $w \in \mathcal{L}(G)$. Let $NECESSARY_{\text{CFG}} = \{\langle G, A \rangle \mid A \text{ is a necessary variable in } G\}$.
 - Show that $NECESSARY_{\text{CFG}}$ is Turing-recognizable.
 - Show that $NECESSARY_{\text{CFG}}$ is undecidable.
- (20 points) Show that $ALL_{\text{DFA}} = \{\langle M \rangle \mid M \text{ is a DFA and } \mathcal{L}(M) = \Sigma^*\}$ is in **P**.