CS:4330 Theory of Computation **Spring 2018**

HW 8: Time Complexity: P, NP, NP-completeness

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1. (20 points) Show that NP is closed under union and concatenation.
2. (20 points) Call graphs G and H **isomorphic** if the nodes of G may be reordered so that it is identical to H. Let

   $$ISO = \{\langle G, H\rangle \mid G \text{ and } H \text{ are isomorphic graphs}\}$$

   Show that $ISO \in \mathbf{NP}$.
3. (20 points) Let

   $$MODEXP = \{\langle a, b, c, p\rangle \mid a, b, c \text{ and } p \text{ are positive binary integers such that } a^b \equiv c \pmod{p}\}$$

   Show that $MODEXP \in \mathbf{P}$. (Note that the most obvious algorithm doesn't run in polynomial time. Hint: try it first where b is a power of 2.)
4. (20 points) Show that if $\mathbf{P} = \mathbf{NP}$, then every language $A \in \mathbf{P}$, except $A = \varnothing$ and $A = \Sigma^*$, is NP-complete.
5. (20 points) We generally believe that *PATH* is not NP-complete. Explain the reason behind this belief. Show that proving *PATH* is not NP-complete would prove $\mathbf{P} \neq \text{NP}$.