Numerical tensor $T \in \mathbb{R}^{n \times 1 \times D}$

Numerical tensors associate $n$-tuples with numerical values:

{ (week 1, Botafogo, 3toques) } $\rightarrow$ 6 retweets  
{ (week 1, Botafogo, 4everton) } $\rightarrow$ 5 retweets  
{ (week 45, Corinthians, espnagora) } $\rightarrow$ 185 retweets  
{ (week 49, Goiás, milton_neves) } $\rightarrow$ 100 retweets  
{ (week 49, Vasco, zigug28) } $\rightarrow$ 1 retweet

Fuzzy relation $R \subseteq [0, 1]^{n \times 1 \times D}$

To define patterns covering $n$-tuples associated with large numbers (for instance), the numerical tensor is turned into a fuzzy relation. E.g., a logistic function is applied:

$$
\begin{align*}
\text{Heaviside step function (threshold } &= 20) \\
\text{logistic function (th. } &= 20, \text{ growth rate } = 0.2)
\end{align*}
$$

The extremal Heaviside step function is commonly used but implies a loss of information w.r.t. the numerical tensor.

Definition of a valid pattern

Given $n$ noise tolerance thresholds $\epsilon = (\epsilon_i)_{i=1..n} \in \mathbb{R}^n$, the pattern $X = (X_i)_{i=1..n} \in \prod_{i=1}^n \mathbb{R}^2$ is valid if and only if:

- $C_{\text{connected}}$: $\forall i = 1..n, \forall x \in X_i, \sum_{t_{R \times X_i} \times (x \times x)} \epsilon_i \leq \epsilon_i$
- $C_{\text{closed}}$: $\forall i = 1..n, \forall x \in D \setminus X_i, \left(\sum_{t_{R \times X_i} \times (x \times x)} \epsilon_i \leq \epsilon_i \right)$
  or
  $\exists x \in U \setminus X_i \left(\sum_{t_{R \times X_i} \times (x \times x)} \epsilon_i \leq \epsilon_i \right)$

Any other relevant property is enforced via an additional user-defined constraint $C$. To have it prune the pattern space, each occurrence of the pattern in its expression must behave monotonically or anti-monotonically. E.g., this constraint forces the patterns, with a numerical dimension $X_i \subset \mathbb{R}$, to have those elements $\tau$-close from each other:

$$
\forall i \in [\min(X_i), \max(X_i)], \exists \tau \in X_i, \text{s.t. } |i - \tau| \leq \tau
$$

Traversal of the pattern space

A binary tree structures the pattern space. At every node, root of a sub-tree:

- $U \subseteq \prod_{i=1}^n \mathbb{R}^2$ contains the elements that must be present in every pattern at the leaves of the sub-tree;
- $V \subseteq \prod_{i=1}^n \mathbb{R}^2$ contains the elements that can be present in some patterns at the leaves of the sub-tree.

At the root, $(U, V) = (\emptyset, \prod_{i=1}^n \mathbb{R}^2)$. A reduced tree, which does not miss any valid pattern, is traversed depth-first. If $V \neq \emptyset$, $\epsilon \in V$ is freely chosen and these rules applied:

- $C_{\text{closed}}$ and $C$ are enforced at every node (pruning).

Quality of the pattern collection ($R$ is $16 \times 16 \times 16$)

<table>
<thead>
<tr>
<th>Per-element tolerance</th>
<th>Per-pattern tolerance (DCE)</th>
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<tbody>
<tr>
<td><img src="#" alt="Graph" /></td>
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Running time in seconds ($R$ is $16 \times 16 \times 16$)

<table>
<thead>
<tr>
<th>Absolute tolerance</th>
<th>Relative tolerance (DCE)</th>
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