

**PROBLEM SET**  
DISCRETE PROBABILITY  
(BASED ON SLIDE-SET)

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**Necessary reading for this assignment:**

- *Slide-set of Lecture 00.B - Discrete Probability:*
    - *An Introduction to Discrete Probability*
    - *Probability Theory*
    - *Bayes' Theorem*
    - *Expected Value and Variance*
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**Review questions.**

1. (Rosen Review Question 7-2)
  - (a) What conditions should be met by the probabilities assigned to the outcomes from a finite sample space?
  - (b) What probabilities should be assigned to the outcome of heads and the outcome of tails if heads comes up three times as often as tails?
2. (Rosen Review Question 7-3)
  - (a) Define the conditional probability of an event  $E$  given an event  $F$ .
  - (b) Suppose  $E$  is the event that when a die is rolled it comes up an even number, and  $F$  is the event that when a die is rolled it comes up 1, 2, or 3. What is the probability of  $F$  given  $E$ ?
3. (Rosen Review Question 7-4)
  - (a) When are two events  $E$  and  $F$  independent?
  - (b) Suppose  $E$  is the event that an even number appears when a fair die is rolled, and  $F$  is the event that a 5 or 6 comes up. Are  $E$  and  $F$  independent?
4. (Rosen Review Question 7-5)
  - (a) What is a random variable?
  - (b) What are the possible values assigned by the random variable  $X$  that assigns to a roll of two dice the larger number that appears on the two dice? .
5. (Rosen Review Question 7-6)
  - (a) Define the expected value of a random variable  $X$ .
  - (b) What is the expected value of the random variable  $X$  that assigns to a roll of two dice the larger number that appears on the two dice?
6. (Rosen Review Question 7-8)
  - (a) What is meant by a “*Bernoulli trial*”?

- (b) What is the probability of  $k$  successes in  $n$  independent Bernoulli trials?
  - (c) What is the expected value of the number of successes in  $n$  independent Bernoulli trials?
7. (Rosen Review Question 7-9)
- (a) What does the linearity of expectations of random variables mean?
8. (Rosen Review Question 7-11) State Bayes' theorem and use it to find  $p(F | E)$  if  $p(E | F) = 1/3$ ,  $p(E | \bar{F}) = 1/4$ , and  $p(F) = 2/3$ , where  $E$  and  $F$  are events from a sample space  $S$ .
9. (Rosen Review Question 7-13)
- (a) What is the variance of a random variable?

**Exercises.**

10. (Rosen 7.2-11) Suppose that  $E$  and  $F$  are events such that  $p(E) = 0.7$  and  $p(F) = 0.5$ . Show that  $p(E \cup F) \geq 0.7$  and  $p(E \cap F) \geq 0.2$ .
11. (Rosen 7.2-17) If  $E$  and  $F$  are independent events, prove or disprove that  $\bar{E}$  and  $F$  are necessarily independent events.
12. (Rosen 7.2-25) What is the conditional probability that a randomly generated bit string of length four contains at least two consecutive 0s, given that the first bit is a 1? (Assume the probabilities of a 0 and a 1 are the same.)
13. (Rosen 7.2-27(b)) Let  $E$  and  $F$  be the events that a family of 4 children has children of both sexes and has at most one boy, respectively. Are  $E$  and  $F$  independent?
14. (Rosen 7.3-9) Suppose that 8% of the patients tested in a clinic are infected with HIV. Furthermore, suppose that when a blood test for HIV is given, 98% of the patients infected with HIV test positive and that 3% of the patients not infected with HIV test positive. What is the probability that
- (a) a patient testing positive for HIV with this test is infected with it?
  - (b) a patient testing positive for HIV with this test is not infected with it?
  - (c) a patient testing negative for HIV with this test is infected with it?
  - (d) a patient testing negative for HIV with this test is not infected with it?