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THE TELEPHONIC SWITCHING CENTRE NETWORK PROBLEM: FORMALIZATION AND COMPUTATIONAL EXPERIENCE

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The switching centre network problem consists of looking for a topology on the urban street network that minimizes the total cost of cables and subterranean piping infrastructure necessary to link a telephonic centre and its subscribers. A simple version of the real model can be viewed as a mixture of Steiner's problem on graphs and a transhipment problem with a single source. We show that a very good initial solution for the problem can be obtained using Dijkstra's minimum distance algorithm. We discuss the theoretical background and the computational experience concerning a software for microcomputers that uses such initialization strategy and that is running quite well in practice. We present the heuristic that looks for scale economies provenient from trajectory coincidences, a local optimum strategy, and also discuss global optimum strategies which should be tested following recent experience concerning Steiner's problem.

1. Introduction

Consider a directed connected graph $G(N, A)$, where N denotes the set of nodes and A denotes the set of arcs. Suppose we have an origin node α (the local switching centre) and the other nodes of N are partitioned into disjoint subsets $N_s \subset N$ as shown in Fig. 1. Each N_s constitutes a service section s, which has a known demand for telephones. Each service section contains a set $V_s \subseteq N_s$ of candidate nodes to locate the distribution box. We define also $V = \bigcup V_s$.

Suppose we have a fixed cost associated with the choice of any arc, plus a variable cost depending on the flow (number of telephones) served by that arc. The switching centre network problem consists of determining a subgraph of G, $G' = (N', A')$, such that: (i) G' is a tree composed by the origin node α and at least one node of each V_s, (ii) the network cost is minimum, and (iii) the installed capacity in each arc admits flow levels that satisfy the demand stipulated for the service sections.

In other words, the switching centre network problem can be viewed as a generalization of the Steiner tree problem on a directed graph [1]. In fact, if we neglect variable costs at the arcs, and if we have a unique node to locate the distribution box of each service section ($|V_s| = 1$ for every s), then $V \subseteq N$ is the set of points which must be linked by a minimum cost tree that contains a directed path between node α and every member of V. In this sense we are treating a NP-hard problem

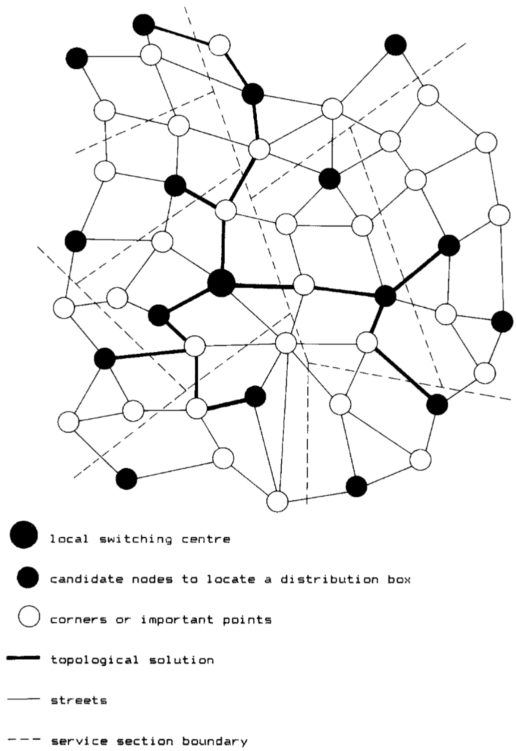

Fig. 1. Local switching centre network.

for which some computational strategies have been devised ([2], [3]). Clearly, when $\{a\} \cup V = N$ the problem reduces to one of finding the minimum spanning tree ([4], [5]). For $|V| = 1$ one needs finding only the shortest path between the origin node α and the destination node of V [6]. In practice, the use of shortest path algorithms has been proved to be a good initialization strategy for the Steiner problem in graphs.

On the other hand, the switching centre network problem can be viewed as an extension of the classical transhipment problem with a single source [7]. For this case, we must neglect the fixed cost associated with the choice of any arc, and also admit a unique node to locate the distribution box of each service section. Since the optimal solution for the transhipment problem is the one that considers the flow from the origin node through the shortest path to every destination node, we have here another good reason to start the computation by using Dijkstra's algorithm. In practice, the interest of such initialization strategy is reinforced by the fact that the relative costs of cables and pipeline infrastructure show that the variable costs in the arcs represent the major part of the total network cost, while a minor part of this total is due to the fixed costs in the arcs.

To transform the switching centre network problem into the Steiner problem or into the single source transhipment problem we must: (i) neglect either the variable cost or the fixed cost associated with any arc, and (ii) admit a unique node to locate the distribution box for each and every service section. Without such assumptions the switching centre network problem can be transformed to the general capacitated network flow problem with concave costs [8], as suggested by Mateus [9]. The dimension and the complexity of the resulting problem has not inspired any computation strategy based on the general facility location problem.

The above analysis presuposes a simplified version of the real problem. The next section presents two integer programming formulations: one for the simple version and another for the complete model that reflects the network planning problem. Section 3 discusses some properties that have inspired the initialization, the heuristics and the local optimum strategies used to implement a software for microcomputer. Section 4 presents a computational experience and digresses on global optimum strategies which could be tried following recent experience concerning Steiner's problem ([1], [3], [14]).

2. Integer programming modelling

2.1. Model requirements

We present two integer linear programming models that synthesize the problem of planning a switching centre network. The first model corresponds to a more simple formulation, that reflects the requirements of minimizing distances, the interest of looking for scale economies provenient from route coincidences, and the need to decide about the location for the distribution boxes. The second model is a more complete formulation, that in addition to the above aspects, specifies the number of pipelines to be installed (at least one for each cable) and considers the scale economies existing in the cost of the cables, due to commercially available cables at different capacity levels. Both models are formulated in terms of the constants and variables described in Table 1, where we indicate the fact that a substancial part

Table 1. Variables and constants of the mathematical models.

VARIABLES		CONSTANTS
p_{ij}: number of subscribers to be served through arc(i,j) (P_{aj} refers to arcs emanating from the origin)	from the simple model	a_{ij}: cost to communicate a subscriber through arc(i,j)
y_{ij}: boolean variable, being 1 if arc ij is chosen for the topology, 0 otherwise		γ_{ij}: fixed cost of the subterranean infrastructure through arc (i,j)
z_k: boolean variable, being 1 if a distribution box is located in node k, 0 otherwise		δ_k: cost of the distribution network when the box is located in node k
		S_s: demand for telephones at the service section s
x_{ij}^m: number of cables for m telephones to be installed in arc(i,j)		c_{ij}^m: cost of a cable for m telephones to be installed in arc(i,j)
q_{ij}: number of pipelines to be installed in arc(i,j) (at least one pipeline for each cable)		b_{ij}: variable cost of the subterranean infrastructure through arc (i,j)

of the constants and variables are common to both models, the complete model having all the variables and all the constants (except a_{ij}) which are present in the simple model.

Although the complete model is more realistic, it is a rather specific and complex model. On the other hand, the simple model presumes quite reasonable assumptions and yet has some parallel in the literature. For these reasons we have used both models in the analysis of the problem. The implementation and the computational results are concerned with the complete model, but the overall strategy has been inspired by the properties evidenced by the simple model.

2.2. Simple model

(1) $$\underset{p,y,z}{\text{Minimize}} \sum_{(i,j)\in A} (a_{ij}p_{ij} + \gamma_{ij}y_{ij}) + \sum_{s} \sum_{k\in V_s} \delta_k z_k$$

subject to:

(2) $$\sum_{(a,j)\in A} p_{aj} = \sum_{s} S_s.$$

(3) $$\sum_{(i,k)\in A} p_{ik} - \sum_{(k,j)\in A} p_{kj} = 0 \quad \text{for every } k\in N-V.$$

(4) $$\sum_{(i,k)\in A} p_{ik} - \sum_{(k,j)\in A} p_{kj} = S_s z_k \quad \text{for every } k\in V_s \text{ and all } s.$$

(5) $$\sum_{k\in V_s} z_k = 1 \quad \text{for all } s.$$

(6) $$p_{ij} < M y_{ij}.$$

(7) The arcs $(i,j)\in A$ such that $y_{ij}=1$ constitute a tree.

The objective function (1) minimizes the total cost resulting from summing up, for each arc, the variable cost of cables and pipelines (function of the number of telephones to be served through the arc), the fixed cost of the subterranean infrastructure, and the cost of the distribution network of each service section.

Constraint (2) assures that the total number of telephones served by the local switching centre is equal to the sum of the demands stipulated for the service sections. Condition (3) requires that the flow arriving at each intermediate node is equal to the flow leaving the node; in particular, not used nodes have null input and output. Condition (4) shows that the difference between the flow in and out of a candidate node is equal to the demand of its service section if it is chosen to locate a distribution box, zero otherwise.

Equation (5) states that for each service section we must select a unique node to locate its distribution box. Constraint (7) asserts that the existence of flow through an arc is conditioned to the choice of the arc for the topology, where M is a big number. Finally, restriction (7) requires that the topology of the network has the configuration of a tree.

2.3. Complete model

(1)′ $$\text{Minimize} \sum_{(i,j)\in A} \sum_{m} c_{ij}^{m}x_{ij}^{m} + b_{ij}q_{ij} + \gamma_{ij}y_{ij}) + \sum_{s} \sum_{k\in V_s} \delta_k z_k$$

subject to (2), (3), (4), (5), (6), (7) and

(8) $$\sum_{m} m x_{ij}^{m} \geq p_{ij},$$

(9) $$q_{ij} > \sum_{m} x_{ij}^{m}.$$

The correspondence between the objective functions of the simple model (1) and the complete model (1)′ is direct. While the simple model accounts for each arc a cost of cables and pipelines which is directly proportional to the number of telephones served through the arc, the objective function of the complete model accounts for each arc the cost of the cables installed in the arc, plus a variable cost for the infrastructure necessary to accomodate the cables.

The restrictions (2), (3), (4), (5), (6) and (7) are common to both models, involving only the variables which are present in the simple model. The main difference in the complete model is to admit slack in the capacity installed in the arc, as implicit in condition (8). Finally, inequality (9) specifies that the number of pipelines is at least equal to the number of cables. In practice, it is usual to leave at least one extra pipeline for future expansion. Furthermore, the number of pipelines is always even, thus making the difference between the numbers of pipelines and cables equal to 1 or 2.

3. Properties and strategies

3.1. Initialization

The fact that every constant cost associated with each arc (i, j) is directly proportional to the distance between i and j shows that the search for shortest paths from the switching centre to distribution boxes is a good initial solution for the problem. If fixed costs γ_{ij} are null and a feasible location vector z is fixed, then the simple model is reduced to a classical transportation problem with a unique source (the local switching centre), whose topological solution is a tree of shortest paths emanating from the centre to the distribution boxes. This property per se justifies the initialization strategy, which consists of finding the shortest paths between the switching centre and the distribution boxes, where the distribution box chosen for each service section is the closest candidate node to the switching centre.

3.2. Heuristics strategy

The function of a module called heuristics is to look for scale economies provenient from trajectory coincidences. In this sense the algorithm corrects some distorsions caused by the shortest path tree solution, trying to use the existent subterranean infrastructure ($\gamma_{ij}=0$), or to force the convergence of near parallel paths. The idea, proposed by Pādua [10], is to fix partial solutions for the initial tree, composed by a certain number of principal chains that link a major quantity of subscribers through the shortest path to the switching centre [11]. In order to link the other subscribers to this partial solution we force the trajectory coincidence through such principal chains, obtaining the shortest path from the location elected for the distribution box of each service section to any principal chain.

3.3. Local optimization rules

The function of the optimization module is to refine the best solutions previously obtained, taking into account the cost data in order to look for scale economies of trajectory coincidences. For this purpose, we have two typical situations that require a network modification:

- It is cheaper to link a certain node of the tree (and all its descendents) through another node of the tree.
- It is cheaper to change the location of the distribution box of a certain service section.

3.4. Spatial pricing

In order to identify some of the situations mentioned in Section 3.3 we may conceive a spatial pricing for the telephones, by extension of the idea suggested by the economic interpretation of the classical transportation problem. The concept can be generalized for the complete model [12], but we stick to the simple model, for which one can establish cost descent network changes. Given a solution p, y, z, we define the cost of a subscriber through arc (i, j) as:

$$(10) \qquad \beta_{ij} = (a_{ij}p_{ij} + \gamma_{ij}y_{ij})/p_{ij}.$$

Based on such value, for each arc and for every node belonging to the topological solution y, we define the unitary cost to link node k to the centre as:

$$(11) \qquad \pi_k = \sum_{ij \in C_{0k}} \alpha_{ij}$$

where C_{0k} indicates the chain between the local switching centre and node k.

With these formulae we can define the price of telephone at service section s as:

$$(12) \qquad \tau_s = \pi_{k_s} + \delta_{k_s}/S_s$$

where k_s is the node of the tree where the distribution box of the service section s is located (i.e., $k_s \in V_s$ and $z_{k_s} = 1$).

A first property of the spatial prices τ_s is that they account values for the telephones in the service sections whose sum corresponds to the total cost of the subterranean switching centre network plus the distribution networks of the service sections.

The spatial pricing concept can be used to identify situations where the cost of a certain tree configuration can be decreased. Fig. 2 shows the case of a service section whose distribution box k can be linked to the local switching centre through the arc (k, j) or the arc (k, i). Assuming unchanged links to other service sections, the choice of the arc (k, j) will be more economical if the unitary cost π_k to link node k is smaller for this alternative. In this case linking of k through j not only diminishes the telephone price in the service section of node k, but also diminishes the telephone price in the service section of nodes m and n (since the fixed cost of the chain (i, j) will also be divided by the subscribers served by k), while the telephone price in the service section of node l is maintained unchanged. Thus the link of k across j leads to a configuration with smaller cost, since no spatial price is increased, while some prices are diminished.

The idea of using spatial values to know whether a terminal node can be better linked through another node of the tree suggests an analog procedure to identify the best linkage of a generic node of the tree, as illustrated in Fig. 3. In this case, if $\pi_i + \beta_{ij} < \pi_j$, then the link of j through i diminishes the telephone price in every service section linked through j or i, but will increase the telephone price in each service section that remains linked through node k. Because of this price increase we are not able to anticipate whether the total network cost will diminish. In the general case the pricing tests are only an indicative of possible good changes in the network, and a posterior total cost verification must be made.

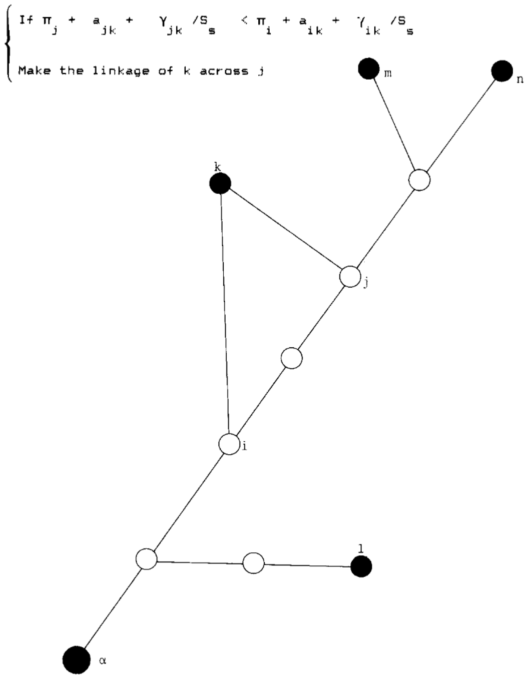

Fig. 2. Case where the pricing test assures the best linkage of a terminal node.

At last, we remember that the same strategy of using spatial values may be used to identify some possible situations where it is more economic to change the location of the distribution box of a certain service section, as illustrated in Fig. 4. The idea consists of linking the service section through a distribution box where the telephone price for the service section is smaller, and a posterior total cost verification is also necessary.

4. Computational results and conclusions

The computational strategies described above were implemented in PASCAL [13] for a 16-bit PC-like microcomputer. The use of the program for the complete model

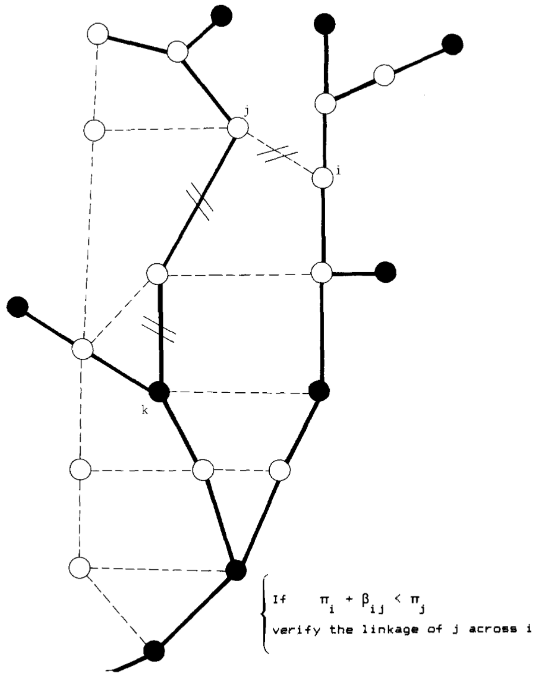

Fig. 3. Case where the pricing test indicates a possibly better linkage of a generic node.

has been quite satisfactory for local switching centre network planning.

Table 2 presents some computational results for the initialization, heuristics and optimization phases. Problem number 1 is a small test problem, while the second and third correspond respectively to medium and large scale practical problems. In the actual implementation the optimization module makes an exhaustive search to verify whether each node of the tree can be better linked through another node of the tree, while the location of the distribution box of each service section is the closest one to the switching centre.

The improvements in the total cost obtained at each phase show how good the initialization strategy is. We conjecture that only minor improvements might be obtained by using global optimum strategies. One possible way to verify our conjecture

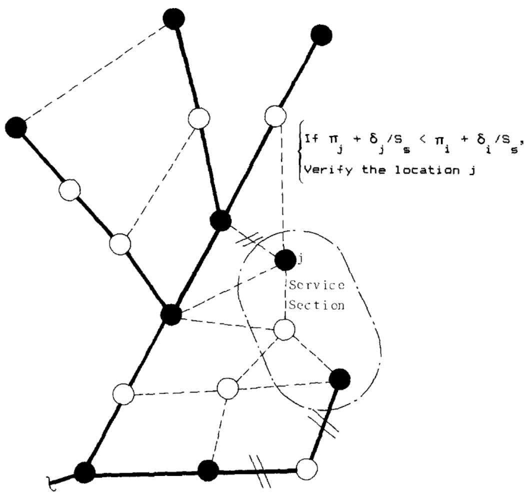

Fig. 4. Case where the pricing test indicates a possibly better location for a distribution box.

might be inspired by recent results concerning the Steiner problem in graphs, such as: the dual ascent approach of Wong [1], the Lagrangean relaxation method of Beasley [14], and the linear programming relaxation approach of Maculan [3]. The adaptation of the later approach seems to be straightforward, leading to linear programs to be solved by generalized variable upper bound methods or by Benders decomposition, which results in simple shortest paths subproblems.

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Table 2. Summary of computational experience.

Problem number	Number of nodes	Number of arcs	Number of service sections	Number of location points	Time of CPU			Number of iterations optimization	Costs (in thousands of Cruzeiros)		
					Initialization	Heuristics	Optimization		Initialization	Heuristics	Optimization
1	18	54	7	9	2.8	4.1	4.4	1	88.057 (100%)	83.476 (94,7%)	83.476 (94,7%)
2	87	228	47	62	19.0	50.1	66.4	4	952.624 (100%)	931.191 (97,7%)	900.719 (94,5%)
3	263	752	117	139	23.5	390.0	908.1	3*	3.543.721 (100%)	3.480.770 (98,2%)	3.437.877 (97,0%)

* The solution can yet be improved.

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