An study of an Edmonds-like property for branching flows

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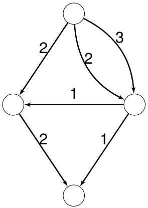
June 10, 2021

Flows in networks

Network $\mathcal{N} = (V, A, u)$:

$D = (V, A)$ with capacity function $u : A \to \mathbb{Z}_+$.
We denote $n = |V|$.



Flow x on $\mathcal{N}$:

$x : A \to \mathbb{Z}_+$ such that $x_{vw} \leq u_{vw}$, $\forall vw \in A$.

Balance vector of a flow x:

$b_x : V \to \mathbb{Z}$ given by
$b_x(v) = \sum_{vw \in A} x_{vw} - \sum_{zv \in A} x_{zv}$.

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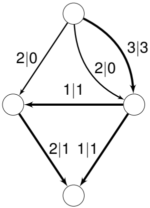

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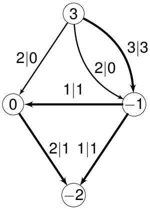

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The flow problem

Given $\mathcal{N} = (V, A, u)$ and a prescribed balance vector b, can we decide if $\mathcal{N}$ has flow x s.t. $b_x(v) = b(v), \forall v \in V$?

Ex:

- (s, t)-flow: $0 \leq b_x(s) = -b_x(t)$ and b is 0, $\forall v \in V \setminus \{s, t\}$.
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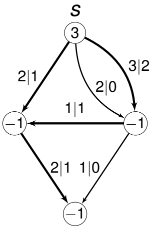

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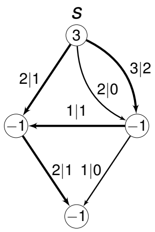

Polynomial-time solvable.

Arc-disjoint flows

Two flows x, y on $\mathcal{N}$ are arc-disjoint if $x_{vw} \cdot y_{vw} = 0\,, \forall vw \in A$.

[Bang-Jensen and Bessy, 14]

Given $\mathcal{N}$, can we decide if it has multiples arc-disjoint flows each with a prescribed balance vector?

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NP-complete in general

Previous results on arc-disjoint flows
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Arc-disjoint flows	Required balance vector	Capacity u	Complexity
x, y	$b_x \not\equiv b_y$	$u \equiv 1$	$\mathcal{NP}$-complete
$x_1, \ldots, x_k$	$b_{x_1} \equiv \cdots \equiv b_{x_k}$	$u \equiv 1$	Polynomial
x, y	$b_x \equiv b_y$	$u_{ij} \in \{1, 2\}$	$\mathcal{NP}$-complete
x, y (s, t)-flows	$b(s) = 2, b(t) = -2$ and $b(v) = 0$, for $v \notin \{s, t\}$	$u_{ij} \in \{1, 2\}$*	$\mathcal{NP}$-complete
x, y (s, t)-flows, acyclic $\mathcal{N}$	$b(s) = k, b(t) = -k$ and $b(v) = 0$, for $v \notin \{s, t\}$	any	$\mathcal{NP}$-complete
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x, y branching flows	$b(s) = n - 1$ and $b(v) = -1, \forall v \in V - s$	$u_{ij} \in \{1, 2\}$	$\mathcal{NP}$-complete
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[Bang-Jensen, Havet and Yeo, 16] on branching flows

- $\mathcal{NP}$-complete for $u \equiv k$, for a constant $k \geq 2$.
- Polynomial-time solvable for $u \equiv n - k$, for a constant $k \geq 2$.
- Under ETH, $\nexists$ polynomial algorithm to decide if $\mathcal{N}$ with $n/2 \leq u \leq n - \log(n)^{1+\varepsilon}$ has 2 arc-disjoint branching flows.

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[Bessy, Hörsch, **M.**, Rautenbach, Sau, 21]

- Branching flows of networks with $u \equiv n - k$ is FPT with parameter k.

Complexity of the arc-disjoint branching flows problem

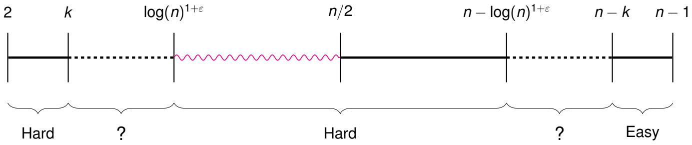

Figure 1: Capacity function

An Edmonds-like property for branching flows

s-(out-)branching: tree s.t. $\forall v \neq s$, $d^-(v) = 1$.

[Edmonds, 73]

A digraph $D = (V, A)$ with $s \in V(D)$ has k arc-disjoint s-branchings if and only if

$$d_D^-(X) \geq k, \forall \emptyset \neq X \subseteq V - s.$$

[Bang-Jensen and Bessy, 14]

Let $\mathcal{N} = (V, A, u \equiv n-1)$. Then $\mathcal{N}$ has k arc-disjoint s-branching flows if and only if

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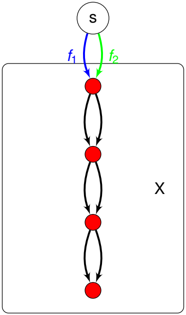

Figure 2: $u = n - 1 = 4$, $k = 2$

An Edmonds-like property for branching flows

- For the existence of an s-branching flow in $\mathcal{N} = (V, A, u \equiv \lambda)$, for $X \subseteq V - s$, we need at least $\left\lceil \frac{|X|}{\lambda} \right\rceil$ arcs entering on it.

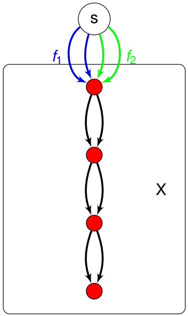

Figure 3: $u = 3$, $k = 2$

An Edmonds-like property for branching flows

Conjecture 1

Let $\mathcal{N} = (V, A, u \equiv \lambda)$. Then, for all $1 \leq \lambda \leq n - 1$, $\mathcal{N}$ has k arc-disjoint s-branching flows if and only if

$$d_D^-(X) \geq k \left\lceil \frac{|X|}{\lambda} \right\rceil, \forall \emptyset \neq X \subseteq V - s. \qquad (1)$$

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Property (1) is always necessary;

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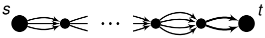
Property (1) is sufficient:

- $\lambda = n - 1$ [Bang-Jensen and Bessy, 14]
- $\lambda = n - 2$
- $\lambda = 1$
- $k = 1$
- D is a multi-path



- D is a collection of multipaths in which we identify s and t.

Arc-disjoint branching flows in multi-branchings

- s-multi-branchings B_s^+: s-branching with parallel arcs.

Conjecture 1 for multi-branchings

Lemma 1

$d^-_{B_s^+}(X) \geq k \left\lceil \frac{|X|}{\lambda} \right\rceil, \forall \emptyset \neq X \subseteq V - s$ in $B_s^+ = (V, A) \Rightarrow$ $\mathcal{N} = (V, A, u \equiv \lambda)$ has k arc-disjoint s-branching flows.

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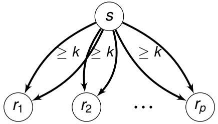

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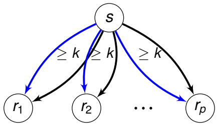

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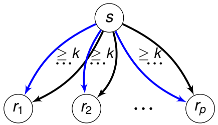

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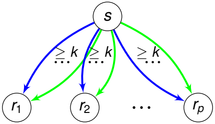

Arc-disjoint branching flows in multi-branchings

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$d^-_{B^+_s}(X) \geq k \left\lceil \frac{|X|}{\lambda} \right\rceil, \forall \emptyset \neq X \subseteq V - s$ in $B^+_s = (V, A) \Rightarrow$
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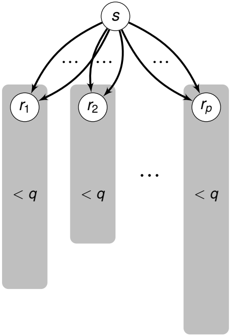
- Induction on the height h of B^+_s:

  h = q



- $d^-(r_i) \geq k \left\lceil \frac{|B^+_{r_i}|}{\lambda} \right\rceil$

A family of counterexamples

Theorem 2

$\forall \lambda \geq 2$ and $\forall k \geq 2$ (even), $\exists\, \mathcal{N} = (V, A, u \equiv \lambda)$ s.t.:

(i) *D satisfies Property (1);*

(ii) *$\mathcal{N}$ doesn't admits k arc-disjoint s-branching flows.*

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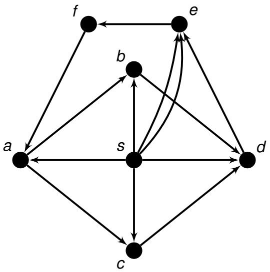

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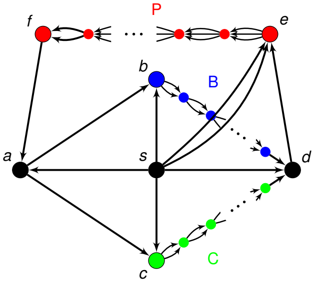

- Subdivide bd, cd and ef $\lambda - 2$ times;
- Arcs of B, C and last arc of P: $\times 2$;
- Other arcs of P: $\times 3$;
- Every arc: $\times k/2$;

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(i) D satisfies $d_D^-(X) \geq k \left\lceil \frac{|X|}{\lambda} \right\rceil, \forall \emptyset \neq X \subseteq V - s$:

- $D[X]$ has a cycle;
- $D[X]$ is acyclic.

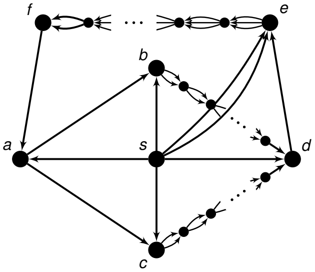

Figure 4: Example for $k = 2$ and $\lambda \geq 2$

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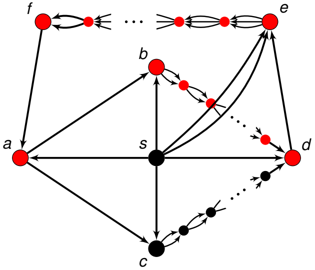

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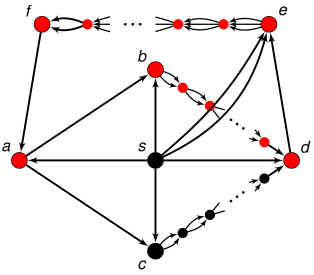

Figure 5: Example for $k = 2$ and $\lambda \geq 2$

- $|X| \geq 2\lambda + 1 \Rightarrow k \left\lceil \frac{|X|}{\lambda} \right\rceil = 6$;

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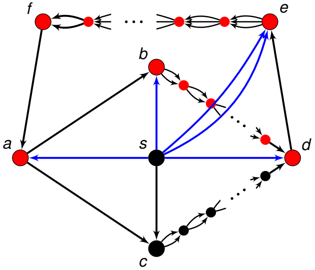

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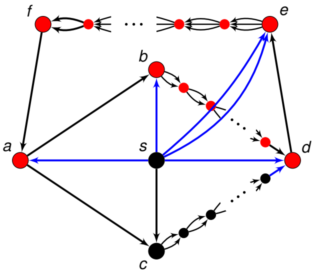

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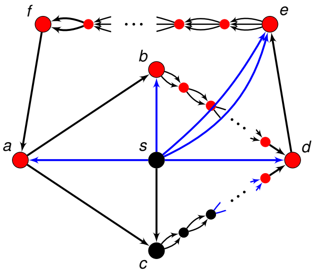

Figure 5: Example for $k = 2$ and $\lambda \geq 2$

- $|X| \geq 2\lambda + 1 \Rightarrow k \left\lceil \frac{|X|}{\lambda} \right\rceil = 6$;

A family of counterexamples

Theorem 2

$\forall \lambda \geq 2$ and $\forall k \geq 2$ (even), $\exists \mathcal{N} = (V, A, u \equiv \lambda)$ s.t.:

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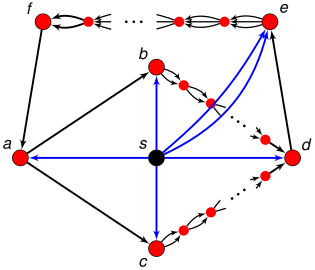

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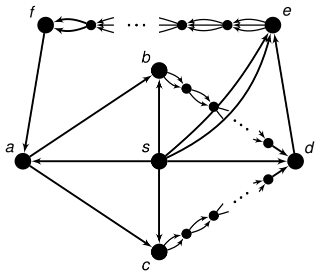

Figure 6: Example for $k = 2$ and $\lambda \geq 2$

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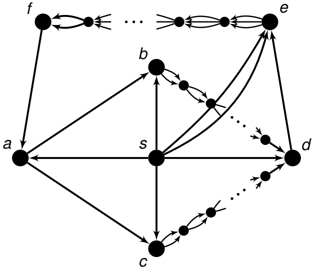

Figure 7: Example for $k = 2$ and $\lambda \geq 2$

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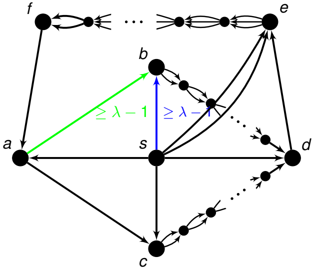

Figure 7: Example for $k = 2$ and $\lambda \geq 2$

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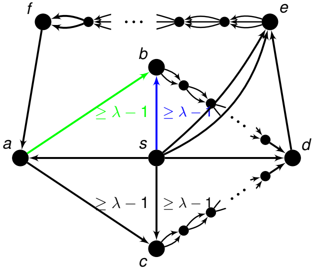

Figure 7: Example for $k = 2$ and $\lambda \geq 2$

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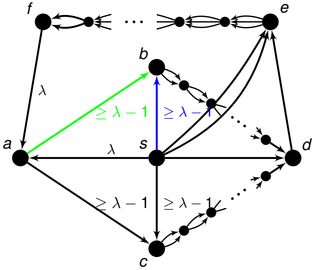

Figure 7: Example for $k = 2$ and $\lambda \geq 2$

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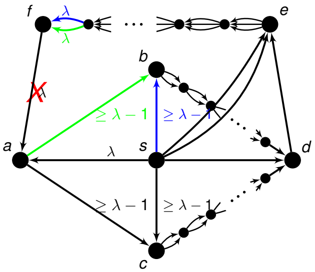


Figure 7: Example for $k = 2$ and $\lambda \geq 2$

Difficulty of finding flows in networks with Property (1)

Theorem 3

It is $\mathcal{NPC}$ to decide if $\mathcal{N} = (V, A, u \equiv \lambda)$ satisfying Property (1) has k arc-disjoint s-branching flows.

3-PARTITION

Input: $S = \{a_1, a_2, \dots, a_{3k}\}$, $\lambda \in \mathbb{Z}^+$, $\lambda/4 < a_i < \lambda/2$, $\sum_{i=1}^{3k} a_i = k\lambda$.
Question: can S be partitioned in k subsets $S_1, S_2 \dots S_k$ so that $\sum_{a_j \in S_i} a_j = \lambda$, $1 \leq i \leq k$?

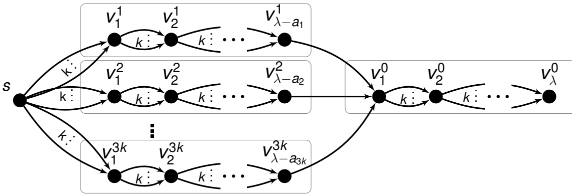
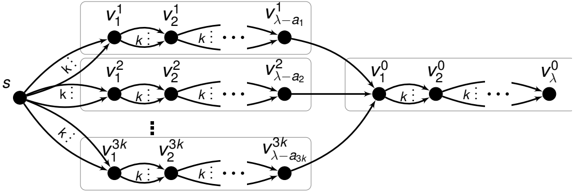


Figure 8: $\mathcal{N}'$

Difficulty of finding flows in networks with Property (1)

- $\exists$ partition of S in $S_1, S_2 \ldots S_k$ s. t. $\sum_{a_j \in S_i} a_j = \lambda \Rightarrow k$ flows on $\mathcal{N}'$



Further research

On arc-disjoint branching flows:

- "Global" condition + "local" condition would be sufficient to guarantee flows?
- Dichotomy between easy and hard cases of DAG's.
- Study the complexity on networks without parallel arcs.
- Study the problem on networks with different capacities and balance vectors.

Obrigada!