Edge coloring: A natural model for sports scheduling

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  Thesis on sport scheduling.
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- Local Search: moves implemented over opponent tables (as done in most of the literature at the time).

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- TTP is a Double Round Robin (DRR) scheduling problem.
- Some other problems and variants consider Single Round Robin (SRR) schedules (for instance, mirrored schedules).
- Here we deal with SRR schedules.

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- The vertices represent the n teams, and each edge $e = (v_i, v_j)$ represents the match in which teams v_i and v_j play against each other.
- A proper edge coloring using $n-1$ colors (a 1-factorization) gives an schedule for the tournament.

Modeling, one-factorization

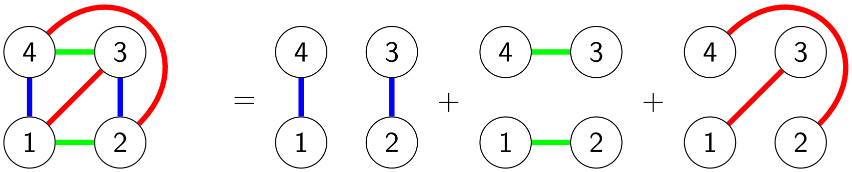

Figure: A tournament represented by a one-factorization of K_4.

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- A neighborhood is connected if it is possible to move from any schedule to any other with a finite number of moves using the neighborhood.
- Here, all neighborhoods will be modeled in terms of operators over one-factorizations (or proper edge colorings with minimum number of colors) of complete graphs.

Team Swap (TS)

Each neighbor in the Team Swap (TS) neighborhood is obtained by swapping two distinct vertices in the graph.

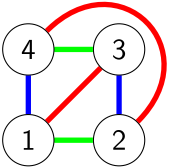

Figure: Team Swap

Round Swap (RS)

Each neighbor in the Round Swap (RS) neighborhood is obtained by swapping the assignment of two distinct colors in the graph.

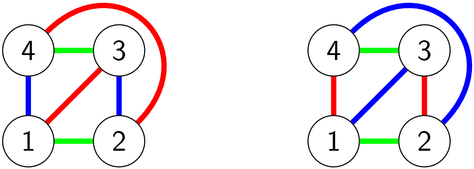

Figure: Round Swap

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- There are 1,132,835,421,602,062,347 nonisomorphic one-factorizations of K_{14}(P. Kaski and P. R. J. Östergård, There are 1,132,835,421,602,062,347 nonisomorphic one-factorizations of K_{14}, Journal of Combinatorial Designs 17, 2009).

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- Each of those neighborhood structures and their union are not connected.

Partial Round Swap (PRS)

In the Partial Round Swap (PRS) neighborhood structure, we select any two distinct colors and consider a cycle in the subgraph generated by the edges colored with those colors. Then, we swap the colors in the cycle to get a neighbor factorization.

Figure: A neighbor obtained with a Partial Round Swap move, by swapping the assignment of colors in one of the cycles.

Partial Team Swap (PTS)

- Consider a color c and two distinct vertices v_1 and v_2, with $c \neq c(v_1, v_2)$.
- Let Ω be a minimum cardinality subset of colors including c in which the set of vertices connected to v_1 by an edge colored with a color in Ω are the same as those connected to v_2 by an edge colored with a color in Ω.
- A neighbor is obtained by swapping the assigment of colors of (v_1, w) and (v_2, w) for each vertex w connected to v_1 or v_2 by an edge colored with a color in Ω.

Partial Team Swap (PTS)

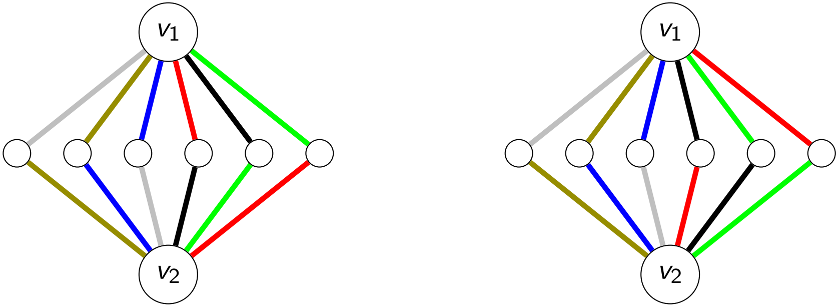

Figure: A neighbor obtained after a Partial Team Swap move.

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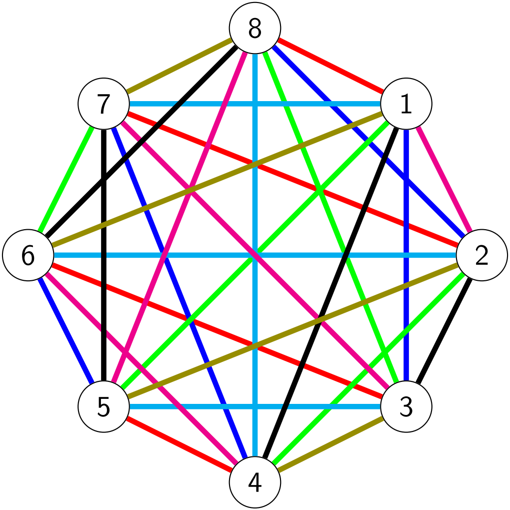
- In contrast to RS and TS, both PRS and PTS neighborhood structures can change the structure of the schedule and they might be connected.
- In the following we investigate the connectivity of these neighborhoods.

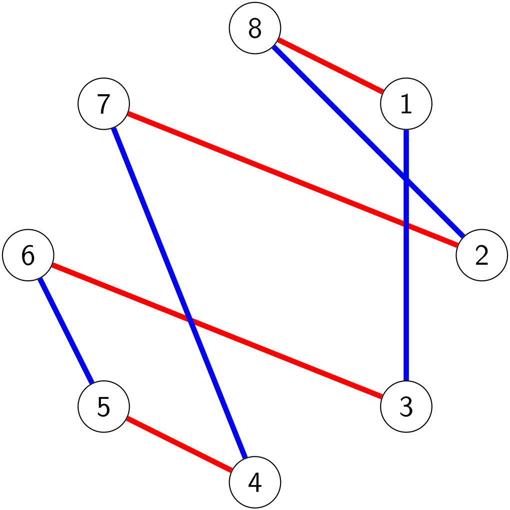
On the connectivity of PRS

- A one-factorization is said to be perfect, called perfect one-factorization (P1F), when the graph generated by the edges of each pair of distinct one-factors is a Hamiltonian cycle.

A P1F of K_8



A Hamiltonian cycle



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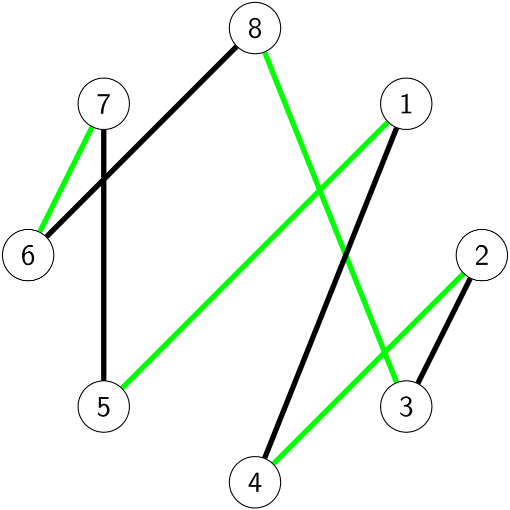

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- A P1F exists for all K_n, where $n \leq 54, n$ even.
- Conjecture: A P1F exists for all K_n, where n is even.

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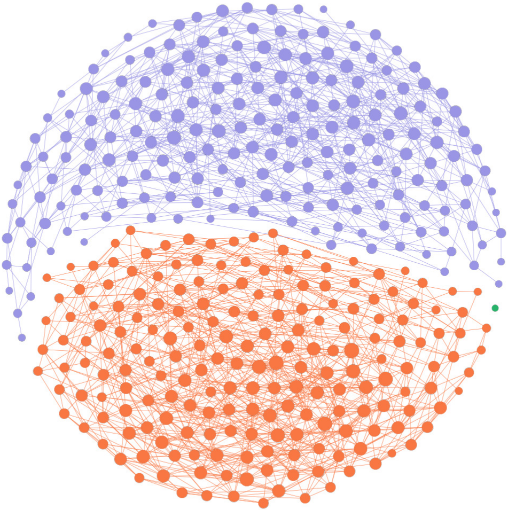
- When a schedule is built by the circle method and n is equal to a prime number plus one, all moves in the PRS neighborhood structure are equivalent to moves in the RS neighborhood structure and the underlying one-factorization is never modified.

On the connectivity of PRS - 1-factorization of K_{10} (396)



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- Are there schedules such that PTS neighborhood is equivalent to TS neighborhood?

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- If the initial solution is constructed using the popular circle method the heuristic would not be able to escape from the canonical coloring.
- To test the impact of this, we programmed the same heuristic starting from the circle method and from a "random" coloring obtained by a method based on a proof of Vizing's theorem.

¿Y ahora que pasa, eh?

Instance	Canonical			Vizing			
	Average	Best	Worst	Average	Best	Worst	Improvement (%)
circ18abal	824.4	812	844	821.6	782	834	0.3
circ18bbal	824.8	810	842	826.2	800	848	-0.2
circ18cbal	820.8	802	842	820.2	802	846	0.1
circ18dbal	827.0	816	836	816.4	802	828	1.3
circ18ebal	819.8	812	836	823.8	804	836	-0.5
circ18fbal	822.4	800	840	822.4	814	832	0.0
circ18gbal	820.2	806	832	819.6	806	842	0.1
circ18hbal	817.0	804	832	816.2	796	842	0.1
circ18ibal	814.4	798	832	823.0	810	850	-1.1
circ18jbal	816.4	804	832	815.8	798	838	0.1
circ18anonbal	836.4	810	858	827.8	818	838	1.0
circ18dnonbal	836.4	822	848	838.8	812	860	-0.3
circ18enonbal	842.4	814	864	839.2	808	862	0.4
circ18fnonbal	837.2	816	856	842.0	828	854	-0.6
circ18gnonbal	823.4	806	842	840.8	816	864	-2.1
circ18hnonbal	849.2	832	870	835.8	822	870	1.6
circ18inonbal	845.8	828	860	847.6	830	866	-0.2
circ18jnonbal	826.2	808	836	819.2	800	850	0.8
Average	828.0	811.1	844.6	827.6	808.2	847.8	0.1

Table 1 Numerical results obtained by the heuristic (18 teams).

¿Y ahora que pasa, eh?

Instance	Canonical			Vizing			
	Average	Best	Worst	Average	Best	Worst	Improvement (%)
circ20abal	1381.0	1360	1396	1135.0	1118	1158	17.8
circ20bbal	1365.8	1340	1396	1134.4	1108	1164	16.9
circ20cbal	1371.6	1348	1390	1119.6	1074	1142	18.4
circ20dbal	1380.8	1362	1392	1140.8	1118	1168	17.4
circ20ebal	1379.0	1364	1396	1130.6	1108	1164	18.0
circ20fbal	1374.4	1362	1388	1125.0	1098	1150	18.1
circ20gbal	1373.8	1350	1386	1123.0	1104	1142	18.3
circ20hbal	1380.4	1350	1410	1131.4	1106	1164	18.0
circ20ibal	1376.6	1366	1392	1129.0	1112	1156	18.0
circ20jbal	1377.4	1370	1384	1121.2	1112	1138	18.6
circ20anonbal	1476.0	1430	1512	1150.0	1124	1188	22.1
circ20bnonbal	1420.6	1396	1442	1154.6	1122	1184	18.7
circ20cnonbal	1441.8	1420	1460	1150.2	1132	1170	20.2
circ20dnonbal	1455.6	1412	1478	1155.2	1122	1172	20.6
circ20enonbal	1477.2	1452	1496	1176.2	1156	1198	20.4
circ20gnonbal	1466.2	1434	1500	1159.6	1122	1192	20.9
circ20inonbal	1400.6	1362	1420	1141.8	1122	1166	18.5
circ20jnonbal	1387.4	1362	1408	1134.4	1116	1148	18.2
Average	1404.8	1380.0	1424.8	1139.6	1115.2	1164.7	18.8

Table 2 Numerical results obtained by the heuristic (20 teams).

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   - Urrutia, de Werra, Januario "Recoloring subgraphs of K_{2n} for sports scheduling", TCS, 2021 (2, 3)

On the connectivity of PTS

- According to the On-Line Encyclopedia of Integer Sequences, our sequence (4, 6, 12, 14, 20, 30, 38, 54 ...) matches the sequence A217948 (the sizes of decks such that a faro shuffle permutation has an $(n-2)$-cycle).

On the connectivity of PTS

- According to the On-Line Encyclopedia of Integer Sequences, our sequence (4, 6, 12, 14, 20, 30, 38, 54 ...) matches the sequence A217948 (the sizes of decks such that a faro shuffle permutation has an $(n-2)$-cycle).
- All numbers listed in A217948 are equal to $p+1$, wherein p is a prime number. (Do you remember the connectivity of RS and PRS?)

What is a faro shuffle?

The faro shuffle, an idealized riffle shuffle, is a term used when a perfect riffle shuffle is performed in such a manner that the deck is split exactly in half and all cards are perfectly alternated.

Faro shuffle

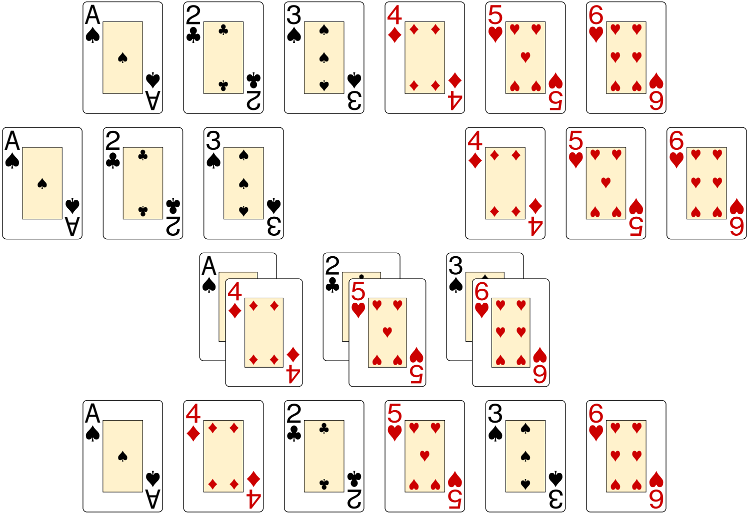

Figure: A faro shuffle with two 1-cycles (A) and (6) and a one 4-cycle (2,4,5,3).

Connecting the circle method and the faro shuffle

- Faro method: scheduling an SRR tournament with faro shuffles.

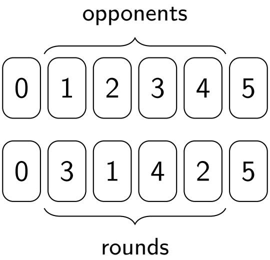

Figure: Scheduling a tournament with $n = 6$ teams based on the faro shuffle of playing cards.

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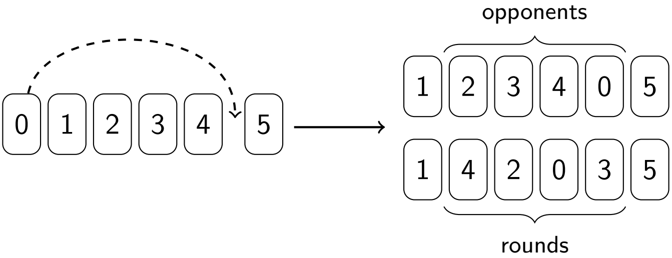

Figure: Scheduling a tournament with $n = 6$ teams based on the faro shuffle of playing cards.

Scheduling an SRR tournament with faro shuffles

We can represent the faro method to schedule a tournament by the following opponent schedule function:

$$\Upsilon(t, r) = \begin{cases} r & \text{if } t = n - 1 \\ n - 1 & \text{if } t = r \\ (2r - t) \mod (n - 1) & \text{otherwise} \end{cases} \quad (1)$$

- Function $\Upsilon(t, r)$ computes the opponent of team t in round r.
- **The faro shuffle and the circle method are equivalent.** (the formula is the same).

Faro lockup

- When constructing an initial schedule with the circle method (faro method), for a subset of the values of $n = p + 1$, for which a faro shuffle permutation of a deck of size n has an $(n-2)$-cycle, the following phenomenon occurs:

Faro lockup

All moves in the PTS neighborhood structure are equivalent to moves in the TS neighborhood structure, all moves in the PRS neighborhood structure are equivalent to moves in the RS neighborhood structure, meaning that the existing neighborhoods do not connect the solution space, moreover they stay trapped in a <u>tiny</u> portion of the solution space.

Januario, Urrutia, de Werra, "Sports scheduling search space connectivity: A riffle shuffle driven approach", DAM, 2016

- A characterization of the values of n for which the canonical one-factorization of K_n is such that for every pair of vertexes u and v the minimum set of vertices that are connected to u and v with the same set of colors has cardinality $n-2$.

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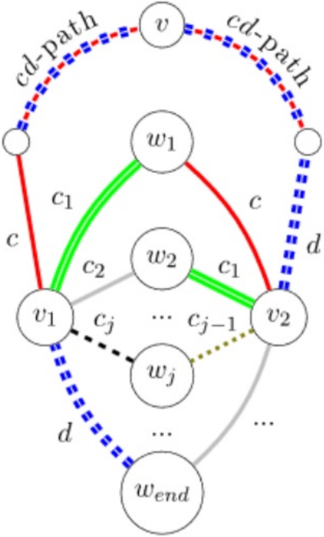
- A characterization of the values of n for which the canonical one-factorization of K_n is such that for every pair of vertexes u and v the minimum set of vertices that are connected to u and v with the same set of colors has cardinality $n-2$.
- **Theorem 1: Given a complete graph K_n with an even number n of vertices, there exist vertices u and v of K_n such that the partition $S_{u,v}$ of $V \setminus \{u, v\}$ generated by the canonical coloring has more than one class if and only if $n-1$ is not prime or the faro shuffle permutation with n elements has no orbit of size $n-2$.**

Januario, Urrutia “A new neighborhood structure for round robin scheduling problems”, C&OR, 2016

- Part of the abstract: **It is known that the neighborhood structures used in those works do not fully connect the solution space. The aim of this paper is to present a novel neighborhood structure for single round robin sport scheduling problems. The neighborhood structure is described in graph theory terms and its correctness is proven. We show that the new neighborhood structure increases the connectivity of the solution space when compared to previous neighborhood structures.**

Januario, Urrutia "A new neighborhood structure for round robin scheduling problems", C&OR, 2016



Januario, Urrutia "A new neighborhood structure for round robin scheduling problems", C&OR, 2016

- Implementation: Best known solution for several instances on two SRR problems.

Januario, Urrutia, Ribeiro, de Werra “Edge coloring: A natural model for sports scheduling”, EJOR, 2016

- Tutorial on the use of edge coloring algorithms for sports scheduling problems.

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- Tutorial on the use of edge coloring algorithms for sports scheduling problems.
- New neighborhood proposal: **A more general move would be to consider edge disjoint alternating chains $C_1, \ldots, C_p$ of even length, possibly larger than two between vertices v_1 and v_2. The set of colors on edges of $C_1, \ldots, C_p$ incident to vertex v_1 is, as in PTS, the same as the set of those incident to v_2. Exchanging colors in each one of the alternating chains gives a new proper coloring that could not be directly obtained with PTS.**

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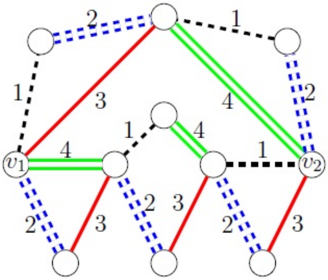

Figure 11: Initial coloring.

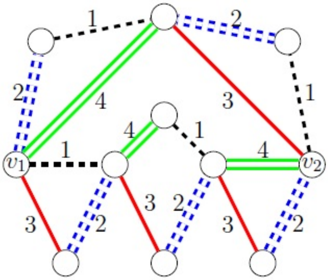

Figure 12: Final coloring.

Januario, Urrutia, Ribeiro, de Werra "Edge coloring: A natural model for sports scheduling", EJOR, 2016

- Recent unpublished paper: **Januario et al. [7] discuss how schedules for compact single-round robin tournaments can be encoded in an edge-coloured complete undirected graph** … **one can select a set of colours (rounds)** $c_0, c_1, \ldots, c_{p-1}$ **that induce** p **disjoint paths** $C_0, \ldots, C_p$ **where path** C_k **is a path from** i **to** j **alternating over the coloured edges** c_k **and** $c_{k+1 (\mod p)}$**. Then, one can re-colour the edges by interchanging the alternating colours on each path. If the paths are disjoint, then the obtained edge-coloured graph is again a valid encoding for a schedule.** … **In the local search part of our procedure, we iterate over the possible disjoint paths, starting by checking** …

Urrutia, de Werra, Januario “Recoloring subgraphs of K_{2n} for sports scheduling”, TCS, 2021

- Highlights:
  - One-factorizations of K_{2n} are the foundation of some sports scheduling problems.
  - We define colorful subgraphs of one-factorizations that allow partial recolorings.
  - Several types of one-factorizations are defined.
  - The presence of the defined subgraphs on each type of factorization is investigated.

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- bichromatic cycles, colorful chordless lanterns and optimally colored even cliques (sub-1-factorizations).
- C-blocking (a.k.a. perfect) 1-factorizations, L-blocking 1-factorizations and K-blocking 1-factorizations.

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- C-blocking $\Rightarrow$ K-blocking

Urrutia, de Werra, Januario "Recoloring subgraphs of K_{2n} for sports scheduling", TCS, 2021

- C-blocking ⇒ K-blocking
- L-blocking ⇒ K-blocking

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- C-blocking $\Rightarrow$ K-blocking
- L-blocking $\Rightarrow$ K-blocking
- C-blocking $\not\Rightarrow$ L-blocking

Urrutia, de Werra, Januario "Recoloring subgraphs of K_{2n} for sports scheduling", TCS, 2021

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- L-blocking $\Rightarrow$ K-blocking
- C-blocking $\not\Rightarrow$ L-blocking
- L-blocking $\not\Rightarrow$ C-blocking

Urrutia, de Werra, Januario "Recoloring subgraphs of K_{2n} for sports scheduling", TCS, 2021

- C-blocking $\Rightarrow$ K-blocking
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- C-blocking $\not\Rightarrow$ L-blocking
- L-blocking $\not\Rightarrow$ C-blocking
- For canonical 1-factorization, L-blocking $\Rightarrow$ C-blocking

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- For canonical 1-factorization, L-blocking $\Rightarrow$ C-blocking
- K_8 has 6 NI 1-factorizations: 2 K-blocking, 1 C-blocking and 0 L-blocking.

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- K_{10} has 396 NI 1-factorizations: 227 K-blocking, 1 C-Blocking and 0 L-Blocking.

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- K_{12} has 526915620 NI 1-factorizations: 5 C-Blocking and 8 L-Blocking. (P. Kaski, A. de Souza Medeiros, P.R.J. Ostergard, I.M. Wanless, "Switching in one-factorisations of complete graphs" Electron. J. Comb., 2014)

Conclusions

- We showed the importance of graph theory, in particular 1-factorizations of complete graphs, in the design of local search algorithms for round robin scheduling problems.

Conclusions

- We showed the importance of graph theory, in particular 1-factorizations of complete graphs, in the design of local search algorithms for round robin scheduling problems.
- Now a days, the community employs much more theoretical approaches when designing those algorithms when comparing with the situation 10 years earlier.

Edge coloring: A natural model for sports scheduling

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Thank you!